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Two-dimensional steady-state general solution for isotropic thermoelastic materials with applications. II. Green's function for two-phase infinite plane

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ABSTRACT

Green's function for isotropic thermoelastic two-phase infinite plane under a line heat source is established in this paper. By virtue of the fourth compact general solutions in Part I which is expressed in three harmonic functions, six new suitable harmonic functions with undetermined constants are constructed for the two semi-infinite planes of the two-phase infinite plane, respectively. The corresponding thermoelastic field can be obtained by substituting these harmonic functions into the general solution, and the undetermined constants can be determined by compatibility conditions and the equilibrium conditions. Numerical results are given graphically by contours.

1. Introduction

Green's functions and fundamental solutions play an important role in both applied and theoretical studies on the physics of solids. They are basic building blocks of a lot of further works. Green's functions can be used to construct many analytical solutions of practical engineering problems by superposition and are very important in the boundary element method as well as the study of cracks, defects and inclusions.

For isotropic elastic materials, the Kelvin Green's function is well-known [1]. For transversely isotropic elastic materials, Lifshitz and Rozentsveig [2] and Lejcek [3] derived Green's functions using the Fourier transform method. Elliott [4], Kroner [5] and Willis [6] obtained them using the direct method and Sveklo [7] found them using the complex method. Pan and Chou [8] solved Green's function in the form of compact elementary functions. For anisotropic materials, Pan and Yuan [9] and Pan [10] obtained the three-dimensional Green's functions for bimaterials with perfect and imperfect interfaces, respectively. The thermal effects are not considered in the above works.

For isotropic thermoelastic materials, Melan and Parkus [11] presented the Green's functions for a point heat source on the surface of a semi-infinite body. Nowacki [12] presented the fundamental solutions for a point heat source in the interior of an infinite body. Yu et al. [13] studied the thermoelasitc field of a bimaterial with an inclusion by the integration method of Goodier [14]. For transversely isotropic thermoelastic materials, Sharma [15] studied Green's functions of transversely isotropic semi-infinite thermoelastic materials in integral form. By virtue of the general solution of Chen et al. [16], Hou et al. [17,18] constructed Green's function for infinite, semi-infinite and two-phase transversely isotropic thermoelastic materials. Xiong and Ni [19], Xiong et al. [20] received 2D Green's functions for semi-infinite and two-phase transversely isotropic

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electro-magneto-thermo-elastic composite. However, the Green's functions for two-phase transversely isotropic thermoelastic materials cannot be degenerated to the corresponding one for isotropic thermoelastic materials. This is because the general solutions for these two kinds of material are in different forms. In additions, Berger and Tewary [21] and Kattis et al. [22] obtained the two-dimensional Green's functions for anisotropic thermoelastic materials.

For interface stress problem, Petrova and Herrmann [23] presented relations for the complex potentials of the thermoconductivity and the thermoelasticity problems for a bimaterial with an internal crack. These complex potentials are expressed in terms of the solution for the same crack in an infinite homogeneous material and constants of the materials. Dundurs and Comninou [24] obtained the Green's function for the interior contact problem through the governing integral equations on the contact zones and to match derivatives of normal displacements. For the interface condition, the perfectly bonded interface is considered in [23,24], while the case of smooth contact interface is not considered.

In this paper, the Green's function for isotropic thermoelastic two-phase infinite plane under a line heat source is presented in form of elementary functions. Two cases of perfectly bonded interface and smooth contact interface all are considered. By virtue of the fourth compact general solution in Part I, six newly found harmonic functions are constructed in terms of elementary functions with undetermined constants. The unique thermoelastic field can be obtained by substituting these functions into the general solutions after determining the constants by compatibility conditions on interface $z = 0$ and the equilibrium conditions of a rectangle of $a_1 < z < a_2$ ($a_1 < 0 < a_2$) and $-b \leqslant x \leqslant b$, where a_1, a_2 and b are arbitrary but should include the source line. Numerical examples are presented. All stress components are shown graphically by contours. Finally, the paper is concluded.

2. Green's functions for two-phase infinite plane under a line heat source

Consider a line heat source H applied in the interior of a two-phase isotropic thermoelastic plane with the interface $z = 0$ (Fig. 1), a 2D Cartesian coordinate (x,z) can be chosen and the line heat source acts at the line $(0,h)$. Based on the general solutions (25-Part I, 27-Part I), the coupled field in this two-phase thermoelastic plane is derived in this section.

2.1. Green's function for two-phase infinite plane with a perfectly bonded interface

Assume that the thermoelastic material in two half-planes $z \geqslant 0$ and $z \leqslant 0$ are perfectly bonded, thus the compatibility conditions on the interface $z = 0$ are in form of

$$u_x = u'_x, \quad u_z = u'_z, \tag{1a}$$

$$\tau_{zx} = \tau'_{zx}, \quad \sigma_z = \sigma'_z, \tag{1b}$$

$$\theta = \theta', \quad \beta\partial\theta/\partial z = \beta'\partial\theta'/\partial z, \tag{1c}$$

where primed quantities refers to the variables in the half-plane $z \leqslant 0$ and the un-primed quantities refer to those in the half-plane $z \geqslant 0$. β and β' are the coefficients of heat conduction of the two half-planes.

For future reference, a series of denotations are introduced as follows:

$$\bar{z}_h = z - h, \quad \bar{r} = \sqrt{x^2 + \bar{z}_h^2}, \quad z_h = z + h, \quad r = \sqrt{x^2 + z_h^2}. \tag{2}$$

Introduce the following harmonic functions for half-plane $z \geqslant 0$.

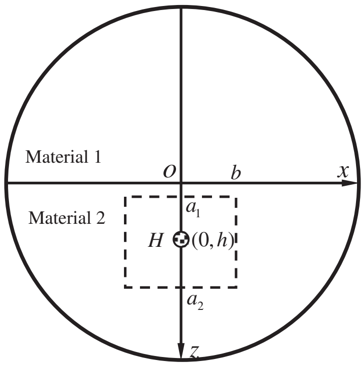

Fig. 1. Thermoelastic two-phase infinite plane having a line heat source of strength H in steady state.