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Short communication

Approximate self-similar solutions for the boundary value problem arising in modeling of gas flow in porous media with pressure dependent permeability

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ABSTRACT

The flow of a gas through porous medium is considered in the case of pressure dependent permeability and viscosity. Approximate self-similar solutions of the boundary-value problems are found.

1. Introduction

The investigation of gaseous flows through tight porous media has drawn huge attention because of its practical importance when dealing with extraction of hydrocarbon gases from unconventional gas reservoirs, such as shale gas and coal bed methane reservoirs. The quantitative analysis of the gas flow may be needed for environmental protection, for example in the areas of contaminant transport and remediation in the unsaturated zone. There is considerable number of results, indicating that the range of applicability of conventional Darcy's law has many limitations due to the nature of the porous medium and peculiarities of the flow regimes [1]. That is why investigation of deviations from Darcy's law becomes a topic of special interest in many studies and applications.There can be distinguished several cases, where the equations, describing gas flow through the porous medium, become non-linear and deviate from the Darcy's law.

Firstly, let us look at the violation of Darcy's law in the case, where gas velocity is relatively high. This class of flows is discussed in publications [2–7]. It has been shown there that, where Reynolds number for gas is big enough, linear correlation between the pressure gradient and velocity is no longer valid, and the nonlinear inertial term in the momentum equation can no longer be neglected. Since we are interested in gas filtration, the nonlinear inertial effects must be considered along with the compressibility one.

The one-dimensional problem for the isothermal gas flow through a porous medium under the conditions of turbulent filtration was studied in [5]. The solution was obtained in terms of cylindrical functions. In [6], these results were extended

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to the case of axisymmetric motion. Analytical solutions for one-dimensional problems of gas flow for quadratic resistance law were found in [7].

In paper [2], the conditions for existence of self-similar solutions for the momentum equations, describing unsteady flow through a porous medium and incorporating the nonlinear inertial term, are examined. According to Pascal [2], self-similar solutions for non-Darcy gas flow exist, if we assume a constant pressure or a constant mass velocity at the outface flow. It should be mentioned here that the interest in self-similar solutions is significant due to the fact that partial differential equations can be reduced to ordinary ones. As it was mentioned in [2], it is always worth searching for self-similar solutions before solving the problem numerically.

Nonlinear effects associated with unsteady flows through a porous medium are analysed in paper [3]. For unsteady flows of polytropic gas and slightly compressible fluid of power law behavior the author derived similar nonlinear parabolic equations for determining pressure distribution. The exact self-similar solutions in a closed form for the first boundary value problem were presented and discussed. The existence of a moving pressure front was established. It has turned out that pressure disturbances in polytropic gas (or slightly compressible fluid of power law behavior) flowing through a porous medium propagate with final velocity, which is a monotonically decreasing function of time.

In paper [4], the authors discuss two classes of non-Darcian flows through porous media, namely the turbulent flow of polytropic gases and the flow of non-Newtonian power law fluids. The momentum equation, governing the gas flow, is the Darcy–Forcheimer equation, while the power law fluid flow model is based on a modified Darcy's law taking into account the non-linear rheological effect on the flow behavior. In both cases the governing equations belong to a class of non-linear degenerate parabolic equations. The solutions of these equations have characteristics of a travel wave. In case, where the boundary conditions are power law functions of time, the authors have managed to found self-similar solutions for the problem.

Another no less important case of deviation from Darcy's law is the situation, where the linear correlation between the pressure gradient and velocity is no longer valid due to the fact, that viscosity or permeability are no longer constants. That is discussed further in this paper. The fact that viscosity depends on pressure has been reported in many publications. We would like to make reference only to some of them, namely the early work by Bridgman [8] and some recent experiments in [9,10]. It should be noted that in general one cannot assume that permeability is independent of pressure.

For a gas flow through tight gas reservoirs the average free path of the gas molecules can no longer be neglected as compared to the average effective radius of rock pore throat. It makes possible for gas molecules to slip along pore surfaces. Such gas slippage phenomenon creates an additional flux mechanism besides viscous flow. This effect yields an overestimated value of permeability of the gas being measured in comparison with the true absolute permeability of a liquid.

The first studies of gas slippage in porous media were conducted by Adzumi [11] and later by Klinkenberg [12], who proved that there exists relationship between the measured gas permeability k and the mean core pressure p:

$$k = k_\infty(1 + d/p).$$

Klinkenberg observed that the gas permeability approaches a limiting value at an infinite pressure p. This limiting permeability k_∞ is sometimes referred to as the equivalent liquid permeability, which is also called Klinkenberg-corrected permeability. The parameter d designates the gas slippage factor, which is a constant, related to the mean free path of the gas molecules at the pressure p and effective pore radius.

The subsequent works have focused on correlating parameters of the Klinkenberg gas slippage factor d and the equivalent liquid permeability k_∞. Basing on the results of their core sample experiments, Heid et al. [13] and Jones and Owens [14] proposed similar approach in the form of correlations between d and $k_\infty : d = C_1 k_\infty^{C_2}$ (where the constants are the following: $C_1 = 11.419,\ C_2 = -0.39$ in [3] and $C_1 = 12.639,\ C_2 = -0.33$ in [14]).

Later Sampath and Keighin [15] studied core samples from a tight gas sand field and proposed a formula relating the gas slippage factor d to the ratio of Klinkenberg-corrected permeability k_∞ to effective porosity $\phi : d = C_1(k_\infty/\phi)^{C_2}$ (corresponding constants are the following: $C_1 = 13.851,\ C_2 = -0.53$). This correlation is interesting since theoretical analysis, reported by Civan [16] and Florence et al. [17] establishes "square root" correlation $(k_\infty/\phi)^{-0.5}$, which is close to $(k_\infty/\phi)^{C_2}$, where $C_2 = -0.53$. As time passed,numerous attempts were made to investigate gas flow in tight porous media.

For example, Javadpour [18] found that the behavior of the gas flow in shales deviates from the behavior described by the Fick's and Darcy's laws. Publication [19] presents results of a laboratory research into the effects of different velocity and pressure testing conditions on a steady-state flow measurements in tight gas sands. The authors also compared measurements of Klinkenberg-corrected permeability using both conventional steady-state technique and unsteady-state permeameters. In the publication [20], intrinsic permeability of sedimentary rocks was measured by using the nitrogen gas and water as pore fluids.

The permeability to the nitrogen gas was 2 to 10 times larger than to water on the same specimen. The permeability to the nitrogen gas decreases with an increase of pore pressure, and the correlation between permeability to the gas and pressure could be described by Klinkenberg's formula for more experimental data.

It has been shown that we need to apply pore pressure above 1 MPa, if we want to avoid Klinkenberg effect. Tight gas and shale gas reservoir systems pose a tremendous potential resource for future development. That is why various attempts have been made to model such kind of flow behavior. Skjetne and Gudmundsson [21] and Skjetne and Auriault [22] theoretically investigated the wall-slip gas flow phenomenon in porous media based on the Navier–Stokes equation, but did not offer any correlation for the Klinkenberg effect. Florence et al. [17] made an attempt to derive a general expression for gas