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Two-machine flow shop total tardiness scheduling problem with deteriorating jobs

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ABSTRACT

This paper studies a two-machine scheduling problem with deteriorating jobs which their processing times depend on their waiting time. We develop a branch and bound algorithm to minimize the total tardiness criteria. A lower bound, several dominance properties and an initial upper bound derived from a heuristic algorithm are used to increase the speed of branch and bound algorithm and decrease its required memory space. Computational results are presented to evaluate effectiveness and efficiency of the algorithms.

1. Introduction

In traditional scheduling systems, it is assumed that processing time of the jobs is constant. However, in the real world there are a lot of factors that influence the processing time and cause it to vary. One of these factors is the waiting time of job before it is processed which can increase the processing time, e.g., in steel rolling or maintenance jobs, any delay in starting time of the job may cause an increase in its processing time. Gupta and Gupta [1] introduced this kind of jobs for the first time. They considered a monotonically increasing function as the deterioration function. For more information about the scheduling problems in which processing time depends on start time see [2,3].

This paper addresses a two-machine flow shop scheduling problem with deteriorating jobs to minimize total tardiness criteria. There are several studies on the flow shop scheduling problem under deterioration in literature. Mosheiov [4] studied multi machine scheduling problem with simple linear deterioration to minimize makespan. He proved that this problem is NP-hard for two-machine. Kononov and Gawiejnowicz [5] also showed that the two-machine flow shop under simple linear deterioration is strongly NP-hard and in three machine flow shop there not exists a polynomial-time approximation algorithm with the worst case ratio bounded by a constant. They also showed that optimal schedule in two and three machine flow shop with proportional processing times of jobs ($P_{ij}(t) = c_{ij}(a + bt)$) is the permutation schedule. Zhao et al. [6] proposed some optimal algorithms for single machine problem under simple linear deterioration. For two-machine flow shop scheduling problem they proved that the optimal schedule can be obtained by Johnson's rule. Wang and Xia [7] studied a no-wait no-idle flow shop scheduling problem under simple linear deterioration. In their problem, some dominating relationships between machines are satisfied. They showed that for the makespan and weighted sum of completion time minimization, the polynomial algorithm exists. Shiau et al. [8] and Wu and Lee [9] considered a two-machine flow shop where the objective

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is to minimize mean flow time. They proposed some lower bound, dominance rules and a branch and bound algorithm to solve the problem. Wang et al. [10] investigated a two-machine flow shop scheduling problem under simple linear deterioration to minimize total completion time. They developed a branch and bound algorithm for the general case and obtained optimal solutions for some special cases. Lee et al. [11] considered a makespan flow shop problem with deteriorating jobs. They proposed an exact algorithm to solve most of the problems up to 32 jobs and a heuristic algorithm to derive the near-optimal solution. Ng et al. [12] studied a flow shop problem with an assumption of existing deteriorating jobs to minimize total completion time. They applied lower bounds and dominance properties to speed up the proposed branch and bound algorithm.

Wang et al. [13] considers single-machine scheduling problems with deteriorating jobs. To minimize makespan with the general linear problem or total weighted completion time with the proportional linear problem, they showed that polynomial algorithms exist in general linear problem. Wang and Xia [14] considered no-wait flow shop scheduling problem with deteriorating jobs. Since this problem is more complicated than the classical one, they showed that polynomial algorithms exist to minimize the makespan. In this paper it is also shown that for the proportional linear problem of minimization of the weighted sum of completion, polynomial algorithms exist, too. The authors also showed when the objective is to minimize maximum lateness or maximum tardiness, the solutions of a classical version may not hold. Wang and Cheng [15] dealt with the machine scheduling problems with the effects of deterioration and learning in single machine problems and flow shop problems. In this model, the processing times of the jobs are defined as functions of their starting times and positions in a sequence. They proposed polynomial solutions for performance measures makespan, total completion time, total weighted completion time, and maximum lateness. Wang et al. [16] investigated single-machine scheduling problems. They showed that this problem is solvable in polynomial time when deterioration and group technology are considered simultaneously. Sun et al. [17] gave some corrections on Cheng et al. [18] for flow shop scheduling problems with deteriorating jobs on no-idle dominant machines. Wang et al. [19] considered the single-machine scheduling problems with a time-dependent deterioration. They developed a mixed integer programming formulation to minimize the total completion time for the problem. In addition to analyze the smallest normal processing time (SPT) worst-case error bound, they proposed two heuristic algorithms. Yang and Wang [20] considered a two-machine flow shop scheduling problem when processing time of a job is a simple linear function of its execution start time. In this study, to minimize total weighted completion time, several dominance properties and two lower bounds for branch-and-bound algorithm are proposed. Huang and Wang [21] showed the parallel identical machines scheduling problems remains polynomially solvable when deteriorating jobs assumption is considered and objective function are the total absolute differences in completion times and the total absolute differences in waiting times. Wei and Wang [22] provided polynomial time algorithms to solve single machine scheduling problems with group technology (GT) and deteriorating jobs. The two objectives of scheduling problems are to minimize the weighted sum of squared completion times and the weighted sum of squared waiting times, respectively. Wang [23] considered a flow shop scheduling problem with deteriorating jobs. They also showed that the makespan minimization problem can be solved in polynomial time for this problem. Wang and Wang [24] show that the single-machine common due- window assignment scheduling problem with learning effect and deteriorating jobs is polynomially solvable.

The real-world application of two-machine scheduling has been investigated by researchers. Lin et al. [25] found that the object ordering problem actually can be formulated as a two-machine flowshop problem. Another real-world application is in the multimedia data object (MDO) scheduling problem. The scheduling of information management systems is becoming an important area of research as pointed out by Pinedo and Chao [26]. Allahverdi et al. [27] showed that a potential model for the MDO scheduling problem is the two-machine flowshop.

According to literature, flow shop scheduling problem with deteriorating jobs and total tardiness minimization has not been studied before and in this paper is going to consider this problem in following sections. Next section is problem definition in which the notations and formulation are described. In Section 3 some dominance properties to eliminate nodes in branch and bound algorithm are proposed. In two subsequent sections, a lower bound is developed and a heuristic algorithm is presented to obtain an initial upper bound for our algorithm. In Sections 6 and 7 solution procedure is presented and the results of the algorithm are evaluated, respectively. Finally, Section 8 is devoted to conclusions and recommendation for future studies.

2. Problem definition

Traditional scheduling problems usually involve jobs with constant, independent processing times. In practice, however, we often encounter settings in which the job processing times vary with time. Hence, there is a growing interest in the literature to study scheduling problems involving start time-dependent processing times (deteriorating jobs), i.e., jobs whose processing times are increasing functions of their starting times. Job deterioration appears, e.g., in scheduling maintenance jobs, steel production, national defense, emergency medicine or cleaning assignments, where any delay in processing a job is penalized by incurring additional time for accomplishing the job [16].

There is a set of n jobs $\{J_1,J_2,\ldots,J_N\}$ which are going to be processed on two flow shop machines $\{M_1,M_2\}$. Job's processing time on each machine depends on its starting time by a linear increasing function considered by Kononov and Gawiejnowicz [5] and Ng et al. [12]:

$$P_{ij}(t) = c_{ij}(a + bt), \tag{1}$$