Applied Mathematical Modelling 38 (2014) 2695–2715

Contents lists available at ScienceDirect

Applied Mathematical Modelling

journal homepage: www.elsevier.com/locate/apm

A mathematical model for horizontal axis wind turbine blades

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ARTICLE INFO

Article history:
Received 13 October 2012
Received in revised form 2 August 2013
Accepted 11 October 2013
Available online 19 November 2013

Keywords:
Wind turbine blade
Mathematical model
Structural damping
Gravitational loading
Aerodynamic loading

ABSTRACT

A mathematical model describing the nonlinear vibration of horizontal axis wind turbine (HAWT) blades is proposed in this paper. The system consists of a rotating blade and four components of deformation including longitudinal vibration (named axial extension), out-of-plane bend (named flap), in-plane/edgewise bend (named lead/lag) and torsion (named feather). It is assumed that the center of mass, shear center and aerodynamic center of a cross section all lie on the chord line, and do not coincide with each other. The structural damping of the blade, which is brought about by materials and fillers is taken into account based on the Kelvin–Voigt theory of composite materials approximately. The equivalent viscosity factor can be determined from empirical data, theoretical computation and experimental test. Gravitational loading and aerodynamic loading are considered as distributed forces and moments acting on blade sections. A set of partial differential equations governing the coupled, nonlinear vibration is established by applying the generalized Hamiltonian principle, and the current model is verified by previous models. The solution of equations is discussed, and examples concerning the static deformation, aeroelastic stability and dynamics of the blade are given.

1. Introduction

To solve a design or control problem of wind turbine blades, a typical way is to reduce the real, complex blade into a simple mechanical model that only depends on important parameters, and then analytical, semi-analytical and numerical methods are employed.

There have been lots of works on the modeling of rotating wind turbine blades. In earlier literature, an isolated blade was usually represented as a rotating rigid body. For example, Chopra and Dugundji [1] established a three-degrees-of-freedom analytical model (including rigid body flap, lead/lag and feather motion) for a rigid blade and studied the nonlinear dynamics of the blade. The blade gets more and more flexible with growing size of wind turbines nowadays. Many flexible system models have been introduced for flexible blades, and the finite element method (FEM) was widely used in modeling. For example, Lee et al. [2] considered horizontal axis wind turbine (HAWT) as a multi-flexible-system composed of rigid subsystems (hub and nacelle) and flexible subsystems (blade and tower), and established a blade model by using the FEM. By using the principle of virtual work in conjunction with the FEM, Baumgart [3] proposed a blade model in which the warping, extension and tilt effects of the cross section were included. Otero and Ponta [4] put forward a Timoshenko beam model for wind turbine blades based on the modified implementation of the variational–asympotic beam sectional technique in the FEM. To deal with problems such as nonlinear dynamics, an analytical model with explicit formulation is convenient. Larsen et al.

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0307-904X/$ - see front matter
http://dx.doi.org/10.1016/j.apm.2013.10.068

[5–7] introduced a variational model for the coupled flap–lead/lag–feather vibration of blades based on the Euler–Bernoulli beam theory. The effect of support point motion was emphasized in this model. Kallesøe [8] proposed an analytical model for blades with pitch action, in which the interactions among gravity, pitch action, varying rotor speed and blade motion were stressed. Ramakrishnan and Feeny [9] introduced a mathematical model for the lead/lag motion of a blade subjected to gravitational and aerodynamic loading, and performed a perturbation analysis for the nonlinear dynamics of the blade in super- and sub-harmonic resonances. Li et al. [10,11] established partial differential governing equations for the coupled extension-flap and coupled extension-lead/lag vibrations, respectively, in which geometric nonlinearity and unsteady aerodynamic loading were highlighted. Li et al. [12] gave a detailed introduction for the rigid body model, flexible system model and aeroelastic stability problem of wind turbine blades. For extensive literature, Houbolt and Brooks [13] and Hodges and Dowell [14] put forward linear and nonlinear models for slender, non-uniform, isotropic helicopter blades, respectively. These two models can be adopted to represent the vibration of slender wind turbine blades directly. Lacarbonara et al. [15] and Arvin et al. [16] proposed an analytical model for the couple extension-flap–lead/lag–feather vibration of elastic, isotropic, non-uniform blades, and dealt with linear modal and nonlinear normal modes problems. These works on analytical models of rotating blades are all for elastic materials.

In early days of wind power industry, blades were usually made of wood, steel beam with fiberglass envelope, or aluminum alloy. With growing size of wind turbines, the requirement for blade materials becomes stricter to get lighter weight, higher strength, more elaborate profile, higher rigidity and more corrosion-resistant skin. The use of composite materials (glass/carbon fiber reinforced plastics, abbr. GFRP/CFRP, bound by epoxy resin) has become an important trend in blade manufacture. Materials and fillers will bring about structural damping for blades. Structural damping plays very important role in the aeroelastic stability of wind turbine blades [12]. Investigations on the structural damping of wind turbine blades or similar structures are chiefly based on the FEM or experimental test, while the research content mainly focuses on damping identification. For example, Coni et al. [17] gave FE and experimental analyses for the structural damping of CFRP beams with tapered boundaries composed of highly damped composite material. Kielb and Abhari [18] proposed an experimental method to test the damping of turbomachinery blades, and discussed the influence of both aerodynamic and structural damping on vibration modes. Chortis et al. [19,20] presented a damped structural dynamics model for large wind turbine blades, in which composite material coupling was highlighted. Li and Law [21] put forward a damping identification procedure for structural system that was based on the sensitivity of acceleration response of the analytical model. Rasuo [22] conducted an experimental test for structural damping of a light helicopter blade made of composite laminated materials. Ding and Law [23] introduced an iterative regularization identification method to determine three types of damping (time-invariant Rayleigh damping, time-variant Rayleigh damping and time-variant modal damping). To the best of authors' knowledge, the effect of structural damping has not been considered in an analytical model for wind turbine blades, and little was done about the influence of structural damping on linear or nonlinear vibration characteristics of blades.

In this paper, an explicit formulation describing the nonlinear vibration of an isolated rotating blade is presented with emphasis on structural damping, gravitational loading and aerodynamic loading. The structural damping induced by materials and fillers is expressed through the Kelvin–Voigt principle approximately, i.e. strain and stress satisfy the relationship $\sigma = E\varepsilon + \bar{\eta}\dot{\varepsilon}$. The equivalent viscosity factor $\bar{\eta}$ can be determined from empirical data, theoretical computation and experimental test e.g. [21–23]. Most effects including coning angle, twist angle, blade root offset from the hub, the eccentricity of the mass center from the shear center of cross section, the centroid bias from the shear center of cross section, the offset between the aerodynamic center and the shear center of blade section are considered, so design and control problems can be solved by using this model or reduced model according to request.

2. General model

Undeformed configurations of a HAWT, a blade and a blade section are shown in Fig. 1. The system consists of a rotating blade and four components of deformation including longitudinal vibration (named axial extension), out-of-plane bend (named flap), in-plane/edgewise bend (named lead/lag) and torsion (named feather). There is a bias e between the center of mass of the hub and the centroid of the blade root, which is caused by the inclined installation of the blade. This bias includes the effect of the hub radius. Each blade section is assumed to be symmetric about the chord line. The center of mass, shear center and aerodynamic center of the blade section all locate on the chord line. e_I denotes the distance from the center of mass to the shear center, it is positive when the center of mass is in front of the shear center. e_A represents the bias between the aerodynamic center and the shear center, it is positive when the aerodynamic center is in front of the shear center. A twist angle θ (=$\theta_0 + \theta_t$, θ_0 is the setting angle and θ_t is the pre-twist angle) between the rotating direction and the chord line is usually designed to get the maximum utilization of wind energy. θ is positive when the plane of rotation is in front of the leading edge.

Several Cartesian right-handed systems (see Figs. 1 and 2) are introduced to describe the configuration of the blade. (xyz) represents a body coordinate system which is rigidly attached to the blade root. The x-axis corresponds to the undeformed elastic axis, the y-axis lies in the plane of rotation and points to the lead/lag direction, and the z-axis is along the flapwise direction. (XYZ) is a global coordinate system with origin at the center of mass of the hub. The Z-axis is aligned with the spin axis backwards, and the Y-axis is parallel to the y-axis. ($X_I\ Y_I\ Z_I$) is an inertial system with origin at the center of mass of the hub. The X_I-axis points upwards, and the Z_I-axis coincides with the Z-axis. For each section of the blade after deformation, a