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Thermoelastic damping effect analysis in micro flexural resonator of atomic force microscopy

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ABSTRACT

In the design of high-Q micro/nano-resonators, dissipation mechanisms may have damaging effects on the quality factor (Q). One of the major dissipation mechanisms is thermoelastic damping (TED) that needs an accurate consideration for prediction. Aim of this paper is to evaluate the effect of TED on the vibrations of thin beam resonators. In particular, we will focus on cantilever beam resonator used in atomic force microscopy (AFM). AFM resonator is actually a cantilever with a spring attached to its free end. The end spring is considered to capture the effect of surface stiffness between tip and sample surface. The coupled governing equations of motion of thin beam with consideration of TED effects are derived. In general, there are four elastic equations that are coupled with thermal conduction equation. Based on accurate assumptions, these equations are simplified and the various boundary conditions have been used in order to validate the computational procedure. In order to accurately determine TED effects, the coupled thermal conduction equation is solved for the temperature field by considering three-dimensional (3-D) heat conduction along the length, width and thickness of the beam. Weighted residual Galerkin technique is used to obtain frequency shift and the quality factor of the thin beam resonator. The obtained results for quality factor, frequency shift and sensitivity change due to thermo-elastic coupling are presented graphically. Furthermore, the effects of beam aspect ratio, stress-free temperature on the quality factor and the influence of the surface stiffness on the frequencies and modal sensitivity of the AFM cantilever with and without considering thermo-elastic damping effects are discussed.

1. Introduction

Micro/nano-scale mechanical resonators are critical components in a wide variety of MEMS applications, such as in accelerometers [1,2], gyrometers [3,4], sensors [5], charge detectors [6] and radio-frequency filters [7–9]. For all these applications, it is important to design and fabricate resonators with very little loss of energy or high quality factors Q, where Q is defined by the ratio of the stored energy in the resonator W and the total dissipated energy per cycle of vibration ΔW

$$Q = 2\pi \frac{W}{\Delta W}. \tag{1}$$

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Understanding the energy loss phenomena in high-Q resonators is a significant topic to optimize their design and performance. The energy loss phenomena in microdevices can generally be classified into two categories: One group is the extrinsic losses, which can be minimized by changing the design or the operating conditions of the MEMS. Air damping and support losses are the main extrinsic losses. The other group is the intrinsic losses, which impose an upper limit on the attainable quality factors. TED and internal losses such as the intergranular losses can be regarded as common intrinsic losses. Intrinsic losses like TED and extrinsic losses are common causes that affect the quality factor. Even though the clamping losses should be minimized by a proper choice of the design of the detection structure [10–12] the TED remains the major dissipation mechanism at room temperature. The TED represents the loss in energy from an entropy rise caused by the coupling between heat transfer and strain rate. Indeed, in a thermoelastic solid with a positive thermal expansion coefficient, the coupling of the strain field to a temperature field provides an energy dissipation mechanism that called TED. In the 1937's, Zener was the first to realize that TED may be a significant dissipation mechanism in mechanical resonators. Zener developed expressions for TED in the form of an approximate one-dimensional (1-D) analysis in thin rectangular beams with flexural vibrations [13,14]. In the Zener's theory, the effect of TED is not taking into account the frequency shift. For this purpose, Lifshitz and Roukes [15] developed thermoelastic equations of a vibrating beam. They solve the TED beam problem of harmonic flexural vibrations of cantilevers by using a one-way coupling approach. They substitute the strain trace with the solution coming from transverse vibration of an isothermal cantilever beam (or clamped–clamped) in the heat transfer equation to obtain the quality factor and frequency shift. Nayfeh and Younis [16] presented an analytical expression to quantify the thermoelastic quality factors of microplates of general shapes. They solved the heat equation for the thermal flow across the microplate so that the thermal equation is decoupled from the mechanical plate equation. Then they utilized a perturbation method to obtain the analytical expression for the thermoelastic quality factor of microplates. Wong et al. [17] derived a closed-form expression for the thermoelastic quality factor of the in-plane vibration of thin silicon rings of rectangular cross-section. They extended the analyses of TED developed by Zener and by Lifshitz and Roukes to cover the in-plane flexural vibration of thin rings. Guo and Rogerson [18], and Sun et al. [19] studied the two-dimensional (2-D) analyses of frequency shifts due to thermoelastic coupling by accounting for heat conduction along the beam thickness and beam span. In both cases, they assume that cubic polynomial [18] and sinusoidal [19] temperature gradients along the thickness direction of the beam for the solution of the coupled thermo-elastic equations of motion for flexural vibrations. Prabhakar et al. [20] presented an exact 2-D analysis of frequency shifts due to thermoelastic coupling for flexural vibrations of a beam. They solved the coupled heat conduction equation for the thermoelastic temperature gradients by considering 2-D heat conduction along the length, and across the thickness, of vibrating Euler–Bernoulli beams. In order to investigate structures with more complex geometries, Hoseinzadeh and Khadem [21] studied thermoelastic vibration and damping of a double-walled carbon nanotube upon interlayer van der Waals (vdW) interaction and initial axial stresses. They applied the Galerkin technique to simplify the governing equation and to obtain the resulting approximate solution. Recently, some efforts have been done in developing finite elements for TED analysis. Guo et al. [22] evaluated the effect of geometry on TED in micro-beam resonators using an eigenvalue formulation and a customized finite element method. They analyzed the vented clamped–clamped and clamped-free beams with square-shaped vents along their centerlines by the finite element method. Their numerical results reveal that the addition of vent sections in the clamped end region can significantly increase the quality factor. Their methodology provides a useful tool in design optimization of micro beam resonators against TED. Also Serra et al. [23] presented a finite element formulation based on a weak form of the boundary value problem for fully coupled thermoelasticity. They pointed out the physical meaning of this dissipation function in the framework of the Biot's variational principle of thermoelasticity. In their finite element formulation two elements were considered: the first was a new 8-node thermoelastic element based on the Reissner–Mindlin plate theory, while the second was a standard three-dimensional 20-node iso-parametric thermoelastic element. For the 8-node element the dissipation along the plate thickness has been taken into account by introducing a through the- thickness dependence of the temperature shape function. With this assumption, the unknowns and the computational effort were minimized. Moreover, Salajeghe et al. [24] considered the nonlinear effect of TED in a microplate. In their study, the difference between linear and nonlinear analysis is shown for calculation of TED. Their results showed that the nonlinear analysis has a significant influence on TED coefficient.

The ability to accurately model and predict energy loss due to the thermoelastic effects is therefore a key requirement in order to improve the performance of high-Q resonators. A cantilever in atomic force microscopy (AFM) is an example of a micro-resonator whose quality factor is limited by TED. However most studies of TED have been based on analytical models, but those are subject to very restrictive assumptions so that they are not sufficiently accurate to predict the behavior of complex three-dimensional (3-D) structures.

The purpose of this paper is to analyze the effects of TED in an AFM resonator beam undergoing flexural vibrations. AFM resonator is actually a cantilever with a spring attached to its free end. The spring is considered to capture the effect of contact stiffness between tip and sample surface. Under small vibration amplitudes, the tip-sample interaction force can be represented by a linear spring. When the sensor tip of the cantilever is in contact with a sample surface, the resonances are shifted in frequency. Furthermore, the thermoelastic coupling generally shifts the vibration frequency. Therefore the resonant frequency and the modal sensitivity analyses are carried out for flexural vibration AFM cantilever with and without considering TED effects here. Various boundary conditions have been considered in order to validate the computational procedure used here. Derivations of the governing equations of motion with 3-D heat conduction are presented in Sections 2. Then, in Section 3, solutions of the governing equations by means of Galerkin method are presented. Finally, in Section 4, this solution procedure is used to compute the frequency shift, quality factor and sensitivity change due to thermoelastic