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Nonlinear electro-dynamic analysis of micro-actuators: Effect of material nonlinearity

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ABSTRACT

This paper presents a nonlinear dynamic analysis of a micro-actuator made of nonlinear elasticity materials. The theoretical formulations are based on Bernoulli–Euler beam theory and include the effects of mid-plane stretching due to large deformation and material nonlinearity. By employing Linstedt–Poincaré perturbation method, the nonlinear governing equation is transformed into a set of linear differential equations which are then solved using Galerkin's method. Numerical results show that the linear constitutive relationship used in previous studies is valid for small deformation only whereas for large deformation, the nonlinear elasticity constitutive relationship must be used for accurate analysis. The effects of initial gap and beam length on the nonlinear electro-dynamic behavior of the micro-actuator are also discussed.

1. Introduction

As reported in previous experimental studies, many materials widely used in micro-electro-mechanical-systems (MEMS) such as metal alloy, single crystal silicon, polycrystalline silicon, silicon oxide, silicon nitride, shape memory alloy, etc. exhibit inherent nonlinear stress–strain constitutive relationship with nonlinear elasticity properties [1–5]. For example, Namazu et al. [5] investigated the thermomechanical deformation behavior of titanium–nickel (Ti–Ni) shape memory alloy (SMA) films and derived their constitutive relationships based on the uniaxial tensile test with in situ X-ray diffraction measurements for a reliable design of Ti–Ni SMA MEMS actuators. It was found from their test results that in some cases the Ti–Ni film exhibits superelastic behavior, and as the percentage of Ti increases precipitation hardening takes place. Their nonlinear stress–strain curves calculated from the constructed constitutive relationships are in close agreement for both the martensite and austenite phases with those obtained from tensile test.

A typical micro-actuator comprises two parallel conducting electrodes, one is the fixed substrate and the other is movable which is modeled as a micro-beam. Extensive experimental and numerical investigations have been conducted on the electrostatic and electro-dynamic behaviors of the micro-beam. Ahmadian et al. [6] developed a geometrical nonlinear finite element model for the dynamic analysis of coupled-domain MEMS devices with electrostatic actuation and squeeze film effect based on Euler–Bernoulli beam theory. Passian et al. [7] presented a micro-cantilever model for an atomic force microscope (AFM) dynamic system to display the useful resonance behavior at kilohertz frequencies. Zook and Burns [8] calculated the natural frequencies of a micro-beam using the finite element method (FEM). Choi and Lovell [9] studied the electro-static bending of a micro-beam numerically by the shooting method. Ahn et al. [10] modeled the electrostatically actuated mi-

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cro-beam as a single degree-of-freedom spring–mass–damper system. Hung and Senturia [11] examined the "leveraged bending" and "strain-stiffening" methods for extending the travel range of electrostatic actuators and experimentally determined the minimum actuation voltage for analog-tuned electrostatic actuators. Chan et al. [12] employed a two-degrees-of-freedom tilting model to describe the translation and rotation motions of capacitors. Zhang and Meng [13] presented a simplified model to study the resonant response and nonlinear dynamics of idealized electrostatically actuated micro-cantilever based devices in MEMS. The equations governing the coupled thermo-elastic dynamic behavior of micro-beams were established by Sun et al. [14] based on the generalized thermoelastic theory. The thermoelastic damping of micro-beam resonators was analyzed by using both the normal mode analysis and the finite Fourier transformation method combined with Laplace transformation. The stabilization of micro-rings and micro-cantilever under an electro-static actuation was discussed by Slava Krylov et al. [15] using parametric excitation. To extend the stable travel range of the electrostatic actuators, the electro-static and electro-dynamic behaviors of curved electrodes, electrodes of specially designed shape and multiple sequentially operated electrodes were investigated by Legtenberg et al. [16], Cheng et al. [17], and Bochobza-Degani et al. [18]. De and Aluru [19] developed relaxation and Newton schemes based on a Lagrangian description of both the mechanical and the electrical domains for the analysis of MEMS dynamics. Yang and his co-workers [20–22] studied the electro-dynamic behavior of electrically actuated geometrically nonlinear micro-beams with or without initial curvature. To the best of authors' knowledge, however, the previous studies, including those mentioned above, are based on simplified linear stress–strain constitutive relationship only and did not account for the effect of the material nonlinearity which has been experimentally observed.

This paper aims to investigate the nonlinear electro-dynamic response of a micro-beam under a step applied voltage, with a particular focus on the influence of the material nonlinearity on the nonlinear dynamic deflection and nonlinear frequency of the beam. The analysis is based on a cubic nonlinear stress–strain constitutive relationship. The nonlinear governing equation is solved by using Linstedt–Poincaré perturbation method combined with Galerkin method. The proposed analysis is validated through a direct comparison with published results. Numerical results are presented in graphical form for clamped micro-beams, showing the effect of material nonlinearity, initial gap and beam length.

2. Nonlinear governing equation

Fig. 1 illustrates an isotropic, homogeneous micro-actuator clamped at both ends modeled as a slender micro-beam subjected to an applied voltage $V(\bar{t})$. Let $p(\bar{t})$ denote the electrostatic force per unit length on the micro-beam, $\bar{u}(\bar{x},\bar{t})$ and $\bar{w}(\bar{x},\bar{t})$ the displacements parallel to the x- and z-axes of the micro-beam as a function of time $\bar{t}$, ρ the mass density, A the cross-section area, L the length, h the thickness, b the width, I the second moment of inertia of the beam. The transverse shear fore, bending moment and the axial force are indicated by Q, M, and T, respectively.

Based on Euler–Bernoulli beam theory, the equilibrium requirements of forces in the vertical direction and the moments of an infinitesimal element of the beam give [23]:

$$\rho A\frac{\partial^2\bar{w}}{\partial\bar{t}^2}+\frac{\partial Q}{\partial\bar{x}}-T\frac{\partial^2\bar{w}}{\partial\bar{x}^2}-p(\bar{t})=0, \tag{1}$$

$$\frac{\partial M}{\partial\bar{x}}-Q=0, \tag{2}$$

$$\frac{\partial T}{\partial\bar{x}}=0. \tag{3}$$

The electrostatic force per unit length of the beam, including the effect fringe field, is over

$$p(\bar{t})=\frac{\varepsilon_0 V^2 b}{2(g-\bar{w})^2}\left(1+0.65\frac{g-\bar{w}}{b}\right), \tag{4}$$

where ε_0 is the vacuum dielectric constant and g is the initial gap between the substrate and the micro-beam.

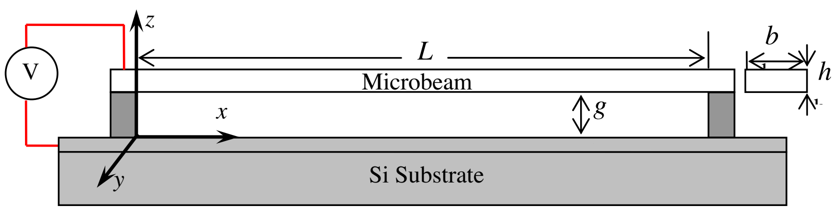

Fig. 1. Clamped–clamped beam model of a micro-actuator.