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An improved nonlocal Gurson model for plastic porous solids, with an application to the simulation of ductile rupture tests

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ABSTRACT

In recent work, the author and co-workers have developed a new methodology to delocalize the damage in Gurson model for porous ductile materials. The motivation was to rectify the difficulties connected to the excessive damage smoothing arising in the practical use of the original damage delocalization method. The new approach consists of delocalizing the logarithm of the damage instead of the damage itself. The relevance of the new method to avoid mesh size effects and satisfactorily reproduce typical ductile fracture experiments are explored in this work.

1. Introduction

The integrity of engineering metal structures under dynamic, cyclic or quasi static loadings requires the study of crack initiation and propagation. The major challenge of such study is to find a predictive tool not only for the initiation of the crack, but also the subsequent extension of the crack. In the case of ductile fracture which is the most dominant failure mode in metals at room or high temperature, the micromechanically based model proposed by Gurson [1] and its heuristic extensions by Tvergaard [2] and Tvergaard and Needleman [3] are widely accepted to describe the three successive stages of ductile fracture process: nucleation, growth and coalescence of cavities. Several studies have demonstrated the performance of the Gurson model to describe crack propagation in pre-cracked metal structures and small uncracked laboratory test samples such as smooth and notched round tensile specimens or plane strain specimens. Among them let mention the works of [3–5] which satisfactorily predict the “cup-cone” fracture phenomenon in smooth axisymmetric tensile specimens, i.e., the well-known fact that in such specimens, the crack first propagates from the axis in radial direction, then deviates at about 45° from the plane when it reaches the vicinity of the cylindrical free surface. Often, the propagation of the crack takes place in regions where the strain and damage localize; these localization regions involve very sharp stress and damage gradients which make the finite element (FE) computations highly mesh dependent: as finer and finer meshes sizes are used, damage and strain have the tendency to localize within bands of zero width. This is a well-known difficulty in modeling ductile materials’ response, which arises as more detailed information about both the deformation and the stress states are demanded in the post localization regime of these materials.

The best way to deal with this difficulty is to add, following an earlier suggestion by Pijaudier-Cabot and Bazant [6], a characteristic length scale to the constitutive model via convolution of the evolution equation of the damage with some weight function. This method, so-called damage delocalization technique, has successfully removed the pathological mesh

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size effects in the FE computations of problems involving ductile fracture, Leblond et al. [7], Tvergaard and Needleman [8,9], Enakoutsa [10] and Enakoutsa et al. [11]. Despite this success, Enakoutsa [10] and Enakoutsa et al. [11] have shown that the damage delocalization technique fails to predict the experimental load–displacement curve for a typical axisymmetric pre-cracked specimen loaded in tension due to the excessive smoothing of the damage in the meshes ahead of the crack tip. They provided a theoretical explanation of this unwanted excessive damage smoothing, following which a simple remedy which consists of delocalizing the logarithm of the damage instead of the damage itself was suggested to deal with it. The agreement obtained between experimental and numerical results was excellent. Despite a recent extension of the logarithmic delocalization technique to a high rate deformation damage model, see [12], the assessment of the new proposal to deal with problems associated with the lack of a characteristic length scale in the Gurson model (unlimited localization of the strain and damage, occurence of bifurcation with infinite number of bifurcated branches, pathological mesh size effects in FE computations) is still at its infancy. The objective of this work is to follow up the study of the assessment of the modified delocalization method. The rest of the paper is organized as follows.

- Section 2 provides the elements of the constitutive relations of the Gurson model and its nonlocal extension.
- Next, Section 3 studies the mesh size effects in the numerical simulations with both the local and nonlocal Gurson models; this study was disregarded in the work of [10,11]. Namely, we show that the addition of a characteristic length scale to Gurson model through damage delocalization allows to eliminate the pathological mesh size effects obtained with the local Gurson model. Nonetheless, as already pointed out by Enakoutsa [10] and Enakoutsa et al. [11] the numerical predictions of the delocalization method are not in good agreement with the experimental results.
- Finally, Section 4 presents an improved version of the nonlocal Gurson model: the logarithmic nonlocal Gurson model. Comparisons of the numerical predictions of the new model and the experiments yield satisfactory results.

2. Gurson model/original and nonlocal versions

The model presented here is called the "Gurson model" for shortness, but in fact incorporates some heuristic improvements brought by Needleman [2] and Gurson [3] to [1]'s original version. These improvements were introduced in order to bring the model predictions to closer agreement with the results of some micromechanical numerical simulations of plastic porous materials, especially during the final phase of coalescence of voids.

Just like the majority of classical plasticity models in large strain, the Gurson model introduces an assumption of additive decomposition of the Eulerian strain rate into elastic and plastic parts, $\mathbf{D} \equiv \mathbf{D}^e + \mathbf{D}^p$, and a hypoelasticity law connecting the elastic strain rate $\mathbf{D}^e$ to some objective time-derivative of the Cauchy stress tensor $\boldsymbol{\sigma}$. We shall not insist on these standard features but concentrate on the new elements of the model. There are four such elements.

The first one is a macroscopic yield criterion for porous plastic media, derived from approximate homogenization of some hollow plastic sphere (typical elementary cell in a plastic porous material) subjected to an arbitrary loading *via* conditions of homogeneous boundary strain rate. This criterion reads

$$\Phi(\boldsymbol{\sigma},\bar{\sigma},f) \equiv \frac{\sigma_{eq}^2}{\bar{\sigma}^2} + 2p\ \cosh\left(\frac{3}{2}\frac{\sigma_m}{\bar{\sigma}}\right) - 1 - p^2 \leqslant 0. \tag{1}$$

In this expression,

- $\sigma_{eq} \equiv \left(\frac{3}{2}\boldsymbol{\sigma}' : \boldsymbol{\sigma}'\right)^{1/2}$ ($\boldsymbol{\sigma}' \equiv$ deviator of $\boldsymbol{\sigma}$) is the von Mises equivalent stress.
- $\sigma_m \equiv \frac{1}{3}\mathrm{tr}\ \boldsymbol{\sigma}$ is the mean stress.
- $\bar{\sigma}$ represents a kind of average value of the yield stress in the heterogeneous metallic matrix, the evolution equation of which is given below.
- p is a parameter connected to the damage (void volume fraction) f through the relation [2,3]:

$$p \equiv qf^*, \quad f^* \equiv \begin{cases} f, & \text{if} \quad f \leqslant f_c, \\ f_c + \delta(f - f_c), & \text{if} \quad f > f_c, \end{cases} \tag{2}$$

where q is "Tvergaard's parameter", f_c the "critical" damage at the onset of coalescence, and δ (>1) a factor describing the accelerated degradation of the material during coalescence.

The second element is a plastic flow rule obeying the normality property, since it was noted by [1] that this property is preserved in the homogenization process, as a consequence of a general theorem in limit-analysis:

$$\mathbf{D}^p = \eta \frac{\partial \Phi}{\partial \boldsymbol{\sigma}}(\boldsymbol{\sigma},\bar{\sigma},f), \quad \eta \begin{cases} = 0 & \text{if} \quad \Phi(\boldsymbol{\sigma},\bar{\sigma},f) < 0, \\ \geqslant 0 & \text{if} \quad \Phi(\boldsymbol{\sigma},\bar{\sigma},f) = 0. \end{cases} \tag{3}$$

The third element is an equation for the hardening parameter $\bar{\sigma}$. This parameter is given by

$$\bar{\sigma} \equiv \sigma(\bar{\epsilon}). \tag{4}$$