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# Dynamic stability of carbon nanotubes reinforced composites

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ABSTRACT

Based on an effective model of multi-walled carbon nanotubes and Donnell-shell theory, an analytical method is presented to study dynamic stability characteristics of multi-walled carbon nanotubes reinforced composites considering the surface effect of carbon nanotubes. From obtained results it is seen that carbon nanotubes composites, under combined static and periodic axial loads, may occur in a parametric resonance, the parametric resonance frequency of dynamic instability regions of CNTs reinforced composites under axially oscillation loading enhances as the stiffness of matrix surrounding CNTs increases, and the surface effective modulus and residue stress of carbon nanotubes make the parametric resonance frequency and the region breadth of dynamic instability of carbon nanotubes reinforced composites increase.



## 1. Introduction

CNTs (carbon nanotubes) have broader application fields as higher strong and stiffness reinforced fibers, good electronic elements, micro-mechanical high frequency oscillators, nano-sensors and nano-actuators [1–5]. Various investigations into the physical, mechanical and electric behaviors of nanotubes in engineering applications have been largely reported [6–8]. Because electronic, transport and mechanical properties of carbon nanotubes could be extremely sensitive to even very small deformation of their otherwise perfect cylindrical geometry, recently considerable attention has turned to stability behavior of carbon nanotubes [9–11].

Generally, there are two theoretical approaches to understand the mechanical behavior of nano-structures such as molecular-dynamics simulations and continuum mechanics. The molecular dynamic (MD) simulations have been given by several investigators [12,13]. However, the computational problem here is that the time steps involved in the MD simulations are restricted by the order of femto-seconds. So even after a million time steps, we can reach only the nanosecond range, in which period most of the mechanical, physical, or magnetic events have not even started [14]. Yakobson et al. [12] introduced an atomistic model for axially compressed buckling of single-walled nanotube and found that all changes of buckling patterns displayed by the molecular-dynamics simulations could be predicted by the continuum shell model. Based on a continuum mechanics model, Wang et al. [15–17] constructed different continuum models to obtain an effective bending modulus of CNTs with various rippling deformations. Wang et al. [11] have recently studied elastic buckling of an individual MWNTs subjected to radial external pressure based on the laminated shells model, and their results shown that the critical pressure predicted by using continuum mechanics model is in reasonably good agreement with the some experiment results from Tang et al. [18], which further offers an evidence that the elastic shell model can be used to study stable behavior of single- or multi-walled carbon nanotubes.

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Most of previous works on stable behavior of carbon nanotubes have mainly focused on the static buckling behavior of CNTs under axial compression loading [10,11] and under bending loading [19,20]. However, when carbon nanotubes are used as AFM tip and various dynamic sensors and actuators etc., the carbon nanotubes under dynamic loading may be occur in dynamic instability [21,10]. To our knowledge, few reports on the dynamic stable behavior of carbon nanotubes embedded in elastic matrix, under combined static and periodic axial loads, is available in literatures. This is apparently due to the fact that experimental research and molecular dynamics simulation for the dynamic stability of carbon nanotubes embedded in elastic matrix remain a formidable task. On the other hand, because the ratio of surface area of nanostructure materials to its volume increases much larger, the surface effects of nanostructure materials can influence the physical and mechanical properties of the nanostructure materials. Therefore, some researches on the surface effect on the physical and mechanical behaviors of carbon nanotubes and nanowires have been done in recent years [22–24]. Thus, for the applications of nanotubes, the study on the dynamic buckling characteristics of carbon nanotubes reinforced composites considering the surface effect of nanostructure materials is of special interest.

In this paper, an analytical method is presented to solve the dynamic buckling of multi-walled carbon nanotubes reinforced composites under axially dynamic loads, utilizing an effective model of multi-walled carbon nanotubes [2], Donnell-shell theory [25] and considering the surface effect of nanotubes. It is seen from calculating examples that MWNTs reinforced composites under an axially oscillation loading exhibit the parametric resonance, and can occur over a range of excitation frequencies even if the static axial loading is tensional loading. It is shown that the dynamic stable characteristic of MWNTs reinforced composites under axially oscillation loading is obviously different on the static bucking of the MWNTs reinforced composites. It is also seen from numerical examples that the parametric resonance frequencies and the region of dynamic instability of MWNTs reinforced composites, under an oscillation axial loading, are not only dependent on the layer number of MWNTs, the stiffness of matrix, the slenderness ratio of MWNTs, the amplitude value of axial excitation loading and the oscillation period, but also dependent on the tensional or compressed state of static component of loading exerted on the MWNTs reinforced composites. Results also show that the surface effective modulus and residue stress of carbon nanotubes make the parametric resonance frequency and the region breadth of dynamic instability of carbon nanotubes reinforced composites increase.

## 2. Theoretical formulations and solving method

The mechanical properties of single- or multi-walled carbon nanotubes have been effectively studied based on elastic shell models [12,15–17]. Motivated by these ideas, carbon nanotubes composites can be taken as a set of concentric cylindrical shells with surrounding polymer matrix which is considered as a spring element defined by Winkler model, as shown in Fig. 1a. A cylindrical coordinate system $(x, \theta, z)$ is taken in the carbon nanotubes composites, where $x$ is the longitudinal direction of tube, $\theta$ is the circumferential direction and $z$ is the radial direction. The surface defined by $z = 0$ is set on the middle surface of carbon nanotube.

Because MWNTs (multi-walled carbon nanotubes) embedded in matrix as shown in Fig. 1a could be described by an effective model of co-axial cylindrical shells with uniform interval spacing $\delta$ [2], The effective cross-sectional area of the MWNTs is written as

$$A_{eff} = 2\pi h\left\{NR_0 + \sum_{n=1}^{N}\delta(n-1)\right\}, \tag{1}$$

where $R_0$, is the non-relaxed radius, as follows:

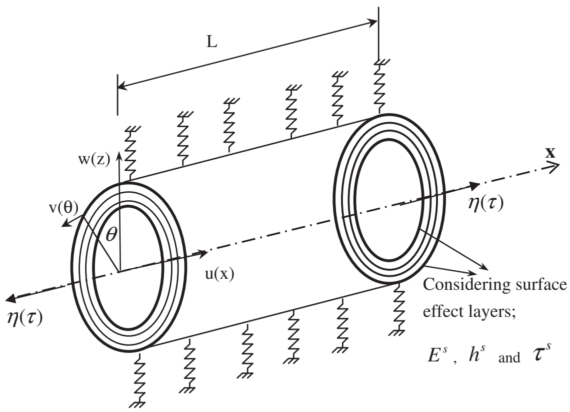


**Fig. 1a.** Model of MWNTs embedded in elastic matrix under an axially oscillating loading, $\eta(\tau) = \eta_0 + \eta_a \cos\overline{p}\tau$ .