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Aggregating crisp values into intuitionistic fuzzy number for group decision making

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ABSTRACT

This paper presents a multiple attribute group decision making model based on aggregating crisp values into intuitionistic fuzzy numbers. First, each alternative is evaluated with respect to their attributes, whose values are provided by decision maker as crisp numbers. Second, to make a reasonable normalization of attribute values in the group decision making environment, a maximum grade and a minimum grade are added to the attribute values. These normalized attribute values are then aggregated (per attribute) into an induced intuitionistic fuzzy number. Each alternative is then evaluated according to the induced intuitionistic fuzzy number. To show the major technical advances in this paper, comparisons with other methods are also made. Finally, an experimental analysis for supplier selection is given to illustrate the reasonableness and efficiency of the introduced method.

1. Introduction

Group decision making (GDM) is a common and crucial human activity, by which a decision maker (DM, or expert) can deal with complex decision problems, involving conflicting and non-commensurable attributes (criteria). GDM is an active area of research at present [1–7].

Generally speaking, there are two main expressions to describe a GDM problem, one is that each DM provides his/her decision information expressed by

$$\begin{array}{r} \\ \text{Alternative } 1 \\ \text{Alternative } 2 \\ \vdots \\ \text{Alternative } m \end{array} \begin{array}{c} \begin{array}{cccc} \text{Attribute } 1 & \text{Attribute } 2 & \cdots & \text{Attribute } n \end{array} \\ \begin{pmatrix} x_{11} & x_{12} & \cdots & x_{1n} \\ x_{21} & x_{22} & \cdots & x_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ x_{m1} & x_{m2} & \cdots & x_{mn} \end{pmatrix} \end{array}, \tag{1}$$

where x_{ij} $(i = 1, 2, \ldots, m,\ j = 1, 2, \ldots, n)$ is attribute value against each alternative. Whose final aim is to rank the alternatives.

Most recent literature represents models in this form. A general procedure for GDM in Eq. (1) requires following some steps [8]:

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(1) Rating phase. Each DM provides his/her individual decision on alternative with respect to attribute.
(2) Normalization phase. The decision information x_{ij} $(i = 1, 2, \ldots, m,\ j = 1, 2, \ldots, n)$ is unified by normalization.
(3) Weighting phase. The weights of attributes and DMs are assigned or calculated.
(4) Aggregation phase. All individual decisions are aggregated into a group opinion.
(5) Selection phase. The grade of each alternative can be calculated and the priority of all alternatives can be determined.

Please refer to [9–14] for details. Good reviews on GDM can be found in [15–19].

Another expression of GDM is that each alternative is evaluated by

$$\begin{array}{c} \\ \text{DM 1} \\ \text{DM 2} \\ \vdots \\ \text{DM t} \end{array} \begin{array}{c} \begin{array}{cccc} \text{Attribute 1} & \text{Attribute 2} & \ldots & \text{Attribute } n \end{array} \\ \begin{pmatrix} x_{11} & x_{12} & \cdots & x_{1n} \\ x_{21} & x_{22} & \cdots & x_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ x_{t1} & x_{t2} & \cdots & x_{tn} \end{pmatrix}, \end{array} \tag{2}$$

where x_{kj} $(k = 1, 2, \ldots, t,\ j = 1, 2, \ldots, n)$ is attribute value against each DM. Whose final aim is also to rank the alternatives.

There are a few studies in above form in the present literature. The goal of the GDM is to aid DMs in integrating objective measurements with value judgments which are not based on individual opinions, but rather on collective group ideas [20]. The key to resolve GDM problems is how to aggregate the values in an attribute/column vector into an overall attribute value, then obtain a collective decision as Eq. (1). This study employs the form in Eq. (2).

Owing to the increasing complexity in modern society, aggregating group's knowledge and experiences to make an appropriate decision is a common method. Furthermore, the aggregating crisp values into intuitionistic fuzzy information is an interesting and important research topic in GDM. For instance, when the satellite remote sensing technology is used in measuring the marine environmental pollution, a lot of data on each attribute are collected everyday. How to analyze these data and to find valuable changes on each attribute are an urgent problem to be solved. Whether these data can be aggregated into a special form of number, for example, intuitionistic fuzzy number (IFN) [21]. This paper will focus on the research.

Atanassov [22,23] introduced the intuitionistic fuzzy set, which is more suitable for dealing with fuzziness and uncertainty than the ordinary fuzzy set developed by Zadeh [24]. Many new approaches and theories treating imprecision and uncertainty have been proposed [25–28].

Considering that (1) many DMs use crisp value to assess alternative with respect to attribute; (2) an IFN is not only straightforward in form, but it also contains a large amount of information; and (3) grading an IFN is simple, in this paper, we intend to aggregate the all individual decisions (per attribute) into an IFN, in which each attribute value is provided by DM, and expressed in crisp value. The aggregated results can form a collective decision matrix, whose attribute values of alternatives are expressed by the induced IFNs.

These are at present only a few methods about the aggregation technique of intuitionistic fuzzy information. Yue [29] and Yue et al. [30,31] developed a technique for aggregating crisp values into an IFN according to the Golden Section. Yue et al. [32] introduced a method for aggregating crisp values into an IFN based on the Minimax Criterion. Yue [33] described an approach for aggregating interval numbers into interval-valued intuitionistic fuzzy information for GDM. Recently, Yue and Jia [34] modeled a method to aggregate crisp values into interval-valued intuitionistic fuzzy information for GDM. Yue [35] proposed a GDM approach based on aggregating interval data into interval-valued intuitionistic fuzzy information.

To make a step forward in the application of the aggregation method, this paper will further enrich the related technology, and focuses on aggregating crisp values into an IFN based on their mean value. In order to do so, the rest of this paper is organized as follows. Section 2 gives a brief description of intuitionistic fuzzy set and related concepts. Section 3 is to devoted to the current method. Section 4 makes the comparisons between the proposed method and other methods. Section 5 demonstrates an experimental analysis to illustrate the reasonableness and efficiency of the introduced method. The final section is conclusion of this paper.

2. Intuitionistic fuzzy set

For convenience, throughout this paper, let $M = \{1, 2, \ldots, m\}$, $N = \{1, 2, \ldots, n\}$ and $T = \{1, 2, \ldots, t\}$; $i \in M$, $j \in N$, and $k \in T$.

An intuitionistic fuzzy set [22] A in a finite set X can be written as

$$A = \{\langle x, \mu_A(x), \nu_A(x)\rangle | x \in X\}, \tag{3}$$

which is characterized by a membership function $\mu_A : X \to [0,1]$ and a non-membership function $\nu_A : X \to [0,1]$ with the condition $0 \leqslant \mu_A(x) + \nu_A(x) \leqslant 1$, where the numbers $\mu_A(x)$ and $\nu_A(x)$ represent, respectively, the membership degree and non-membership degree of the element x to the set A.

A third parameter of intuitionistic fuzzy set is $\pi_A(x) = 1 - \mu_A(x) - \nu_A(x)$, which is known as the intuitionistic fuzzy index or hesitation degree of whether x belongs to A or not. Clearly, $0 \leqslant \pi_A(x) \leqslant 1$ for every $x \in X$. If the $\pi_A(x)$ is smaller, the