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Inhibition or enhancement of chaotic convection via inclined magnetic field

J.M. Jawdat [a], S. Momani [b,c,*], I. Hashim [a,d]

[a] School of Mathematical Sciences, Universiti Kebangsaan Malaysia, 43600 UKM Bangi Selangor, Malaysia
[b] Department of Mathematics, Faculty of Science, The University of Jordan, Amman 11942, Jordan
[c] Department of Mathematics, Faculty of Science, King AbdulAziz University, Jeddah 21589, Saudi Arabia
[d] Solar Energy Research Institute, Universiti Kebangsaan Malaysia, 43600 UKM Bangi Selangor, Malaysia

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ABSTRACT

In this paper, we investigate the onset of convection in a horizontal layer of fluid which is heated from the underside. An inclined magnetic field is applied to the layer. The Galerkin truncated approximations were used to obtain a Lorenz-like model. The nonlinear system was solved by the fourth-order Runge–Kutta method. The results show that the Hartmann number and the angle of inclination of the magnetic field could inhibit or enhance the onset of chaotic convection.

1. Introduction

Thermal convection in a fluid layer is a fundamental paradigm for nonlinear dynamics including instabilities and bifurcations, pattern formation, chaotic dynamics and turbulence. Convection is said to be chaotic if nearby fluid elements typically diverge from each other exponentially in time. A truncated Galerkin expansion can be used to derive a system similar to the famous Lorenz equation [1–3] describing the dynamics.

A sudden transition to chaotic convection from steady for a low Prandtl number case was reported by Vadasz and Olek [4]. The transition is generated by a subcritical Hopf bifurcation which results in a solitary limit cycle. Vadasz [5] explained via local analytical results the appearance of this solitary limit cycle. Vadasz [6,7] employed similar approach for a convection problem in a pure fluid. Vadasz and Olek [8] found transition to chaos via a period doubling sequence of bifurcations for the moderate Prandtl number case. Sheu [9] showed that the fluid–solid interphase heat transfer could affect the route to chaos convection in a porous medium. In the moderate interphase heat transfer and small/moderate porosity-modified conductivity ratio case, a sudden transition to chaos was predicted. Furthermore, chaos occurred through period-doubling route for the weak interphase heat transfer and small porosity-modified conductivity ratio case. The effect of internal heat generation on the onset of chaotic convection in a porous medium for a low prandtl number case was investigated by Jawdat and Hashim [10]. It was found that a uniform internal heat generation could enhance the onset of chaotic convection. The inhibition of chaotic convection in nanofluids was shown possible by Jawdat et al. [11].

The study of magnetic field effects has important applications in physics and engineering. Sankar et al. [12] studied the effect of the directions of magnetic field radial or axial on the buoyancy-driven convection in a vertical cylindrical annulus filled with a low prandtl number electrically-conducting fluid. They found that the convection flow can be suppressed and

* Corresponding author at: Department of Mathematics, Faculty of Science, The University of Jordan, Amman 11942, Jordan. Tel.: +962 77 500326; fax: +962 6 4654061.
E-mail address: s.momani@ju.edu.jo (S. Momani).

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the flow oscillations can be eliminated by the magnetic field, which is more effective when it is perpendicular to the direction of primary flow. Bednarz et al. [13] demonstrated a successful enhancement of convection with the application of a strong magnetic field generated by a super-conducting magnet. They also emphasized that using a strong magnetic field can suppress or invert the usual gravitational convection. The steady free convection in a rectangular cavity filled with a porous medium saturated with an electrically-conducting fluid under the influence of magnetic field was studied by Grosan et al. [14]. They found that the convective heat transfer can be reduced under the effect of the magnetic field, and the horizontal direction of the applied magnetic field is more effective in suppressing the convection flow than the vertical direction for a stronger magnetic field. Nadeem and Akram [15] investigated the peristaltic flow of a Williamson fluid in an inclined symmetric or asymmetric channel under the influence of an inclined magnetic field. They observed that the increase in Hartmann number and the volume flow rate will decrease the velocity profile, while it decreases with an increase in the inclination angle. Further, Nadeem and Akram [16] discussed the effects of partial slip on the peristaltic flow of a MHD Newtonian fluid in an asymmetric channel. They found that the temperature field and the pressure gradient decrease with the increase in slip parameter L, and magnetic field M, while the pressure rise increases in the peristaltic pumping region. a number of researches have discussed the peristaltic flow problems with the influence of inclined magnetic field in Newtonian and non-Newtonian fluids [17–19]. Sathiyamoorthy and Chamkha [20] considered steady, laminar, two-dimensional MHD natural convection within a liquid gallium filled square enclosure in the presence of an inclined magnetic field for different thermal boundary conditions. It was observed that the application of the magnetic field reduces the convective heat transfer rate in the cavity for any angle inclination. Idris and Hashim [21] also showed that delaying convective motion can be made possible via a magnetic field in a porous medium for a low Prandtl number case. Meanwhile, Mahmud and Hashim [22] showed that a constant, vertical magnetic field could suppress or enhance the chaotic convection.

The present work is aimed at extending the work of Mahmud and Hashim [22] to consider the influence of an inclined magnetic field on chaotic convection for a low Prandtl number case. Applying the truncated Galerkin approximation, an autonomous system is obtained and then analysed to study the effects of an inclined magnetic field on the transition to chaos.

2. Problem formulation

A schematic diagram of the problem geometry is shown in Fig. 1. The fluid layer is heated uniformly from below and cooled from above. In addition, the layer is subject to an externally-imposed inclined magnetic field of strength B with angle ϕ. Let x denote the spatial coordinate in the horizontal direction and z be the vertical axis pointing upwards such that $\hat{e_g} = -\hat{e}_z$.

A linear relationship between density and temperature is assumed and can be presented as $\rho = \rho_0[1 - \beta_*(T_* - T_c)]$, where β_* represents the thermal expansion coefficient. Subject to the Boussinesq approximation, the governing equations for an incompressible Newtonian fluid are the continuity equation, the suitably modified form of the Navier–Stokes equation and heat equation respectively,

$$\nabla \cdot V_* = 0, \tag{1}$$

$$\rho_0\left[\frac{\partial V_*}{\partial t_*} + V_* \cdot \nabla V_*\right] = -\nabla p_* + \nu_*\nabla^2 V_* + \rho\vec{g} + J \times B, \tag{2}$$

$$\frac{\partial T}{\partial t_*} + V_* \cdot \nabla T = \alpha_*\nabla^2 T, \tag{3}$$

$$\nabla \cdot J = 0; \quad J = \sigma(-\nabla\varphi + V_* \times B). \tag{4}$$

Here V_* denotes the velocity, t_* is time, T temperature, p_* pressure, ν_* fluid viscosity, α_* thermal diffusivity, J electric current density, φ electric potential, σ electric conductivity and ρ_0 is a reference value of density.

Garandet et al. [23] suggested that the electric potential in Eq. (4) was significantly reduced to $\nabla^2\varphi = 0$ for a 2-D steady-state situation. Since $\partial\varphi/\partial n = 0$, the unique solution is $\nabla\varphi = 0$. This means that the electric field vanishes everywhere.

The following transformations will non-dimensionalize Eqs. (1)–(4):

$$\begin{aligned} &V = \frac{H_*}{\alpha_*}V_*, \quad p = \frac{H_*^2}{\rho_0\alpha_*^2}p_*, \quad \hat{t} = \frac{\alpha_*}{H_*^2}t_*, \\ &T\Delta T_c = T_* - T_c, \quad x = \frac{x_*}{H_*}, \quad y = \frac{y_*}{H_*}, \quad z = \frac{z_*}{H_*}, \end{aligned} \tag{5}$$

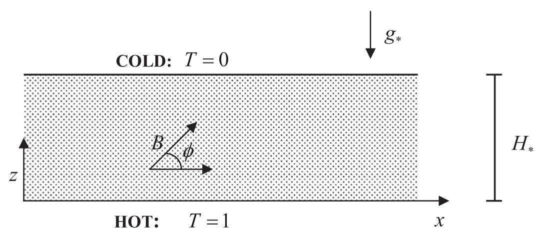

Fig. 1. A schematic representation of the horizontal fluid layer.