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Large deflections of tapered functionally graded beams subjected to end forces

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ABSTRACT

The large deflections of tapered functionally graded beams subjected to end forces are studied by using the finite element method. The material properties of the beams are assumed to vary through the thickness direction according to a power law distribution. A first order shear deformable beam element employed the exact polynomials to interpolate the transverse displacement and rotation, is formulated in the context of the co-rotational approach. The large deflection response of the beams is computed by using the arc-length control algorithm in combination with the Newton–Raphson iterative method. The numerical results show that the formulated element is capable to assess accurately the response of the beams by using just several elements. A parametric study is given to examine the influence of the material non-homogeneity, taper ratio as well as the aspect ratio on the large deflection behaviour of the beams.

1. Introduction

Functionally graded materials (FGMs), which were invented by Japanese scientists in Sendai in 1984 [1], have received much attention from researchers. The FGMs are formed by varying percentage of constituents in any desired spacial direction, and as a result the specific physical and mechanical properties of the formed material can be obtained. FGMs are being used widely as a structural material, and analysis of FGM structures has become an important topic in the field of structural mechanics. A comprehensive list of publications on the analysis of FGM structures subjected to different loadings is given in a review paper by Birman and Byrd [2], contributions that are most relevant to the present work are discussed below.

Sankar [3] proposed an elasticity solution for FGM beams under static transverse loads by assuming the material properties to vary in the thickness direction by an exponential law.Based on the first order shear deformation beam theory, Chakraborty et al. [4] formulated a beam element for analyzing the thermoelastic behaviour of FGM beams by using the exact solution of the governing differential equations of an FGM Timoshenko beam segment to interpolate the displacements and rotation. Using the spectral finite element method, Chakraborty and Gopalakrishnan [5] studied the wave propagation behaviour of FGM beams under high frequency impulse loading. Kadoli et al. [6] formulated a beam element to investigate the static behaviour of metal ceramic beams under ambient temperature by adopting the third order shear deformation beam theory. Taking the warping effect into consideration, Benatta et al. [7] derived an analytical solution to the bending problem of a FGM beam. Singh and Li [8] presented a mathematical model for computing the buckling loads of uniform and non-uniform axially FGM columns. Kang and Li [9,10] proposed closed-form solutions for a nonlinear FGM cantilever beam with the elastic modulus variation in thickness direction under a tip load or a tip moment by deriving an expression

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for the effective bending rigidity. Lee et al. [11] presented a finite element procedure for computing the post-buckling response of FGM plates under compressive and thermal loads. Huang and Li [12] studied the free vibration of non-uniform cross-section beams made of an axially FGM. Employing the multilayering method, Kutiš et al. [13] presented a finite element procedure for modelling a FGM beam with spatial variation of material properties. Also using the finite element method, Alshorbagy et al. [14] investigated the free vibration characteristics of Euler–Bernoulli beams with material graduation in both axially and transversally through the beam thickness. Adopting the Ritz method, Wattanasakulpong et al. [15] investigated the thermal buckling and elastic vibration of the third-order shear deformable FGM beams. Shahba et al. [16] computed the natural frequencies and buckling loads of tapered Timoshenko beams composed of axially FGM by using the exact shape functions of a uniformed homogeneous Timoshenko beam segment to derive the mass and stiffness matrices. Aminbaghai et al. [17] studied the free vibration of FGM beams with continuous spatial polynomial variation of material properties by a fourth order differential equation of the second order beam theory. Using the finite element method, Nguyen et al. [18] studied the dynamic behaviour of non-uniform FGM Timoshenko beams subjected to a variable speed moving load.

Non-prismatic beams with variable cross section are of great important in engineering because of their ability in optimizing the weight and strength of structures. Analytical methods are often encountered difficulties in analyzing the non-prismatic beams due to the presence of variable coefficients in the governing differential equations, and a numerical methods is necessarily employed instead of. In this line of work, Wood and Zienkiewicz [19] computed the large displacement response of a non-uniform column subjected to an eccentric axial compressive force by using the finite element method. In [20], Cleghorn and Tabarrok employed the homogeneous solution of a tapered Timoshenko beam segment to formulate the mass and stiffness matrices for computing the vibration characteristics of the beams. Baker [21] used the weight residual method in solving the governing differential equation of a slender tapered cantilever beam under arbitrarily distributed loads. Lee et al. [22] presented a Runge–Kutta based numerical method to solve the governing differential equations of tapered cantilever beams under large displacements, and then verified the computed results by performing an experiment on a width tapered steel beam. Brojan et al. [23] developed an exact moment–curvature formula for determining deformed shape of non-prismatic cantilever beams obeying the generalized Ludwick law under a tip moment. Attarnejad et al. [24] derived the displacement functions for studying the free vibration of non-prismatic beams by solving the governing equations of motion of a tapered Timoshenko beam element. Shahba et al. [25] introduced the basic displacement functions and then constructed the shape functions for derivation of an efficient 2D beam element for the free vibration analysis of rotating tapered Timoshenko beams.

To the authors' best knowledge, the large deflection of tapered FGM beams subjected to end forces has not been studied in the literature and this topic will be the subject of investigation by using the finite element method in the present work. The material properties of the beams are assumed to be described by a power law distribution through the beam thickness. A nonlinear beam element based on the first order shear deformation beam theory, employing the polynomials obtained from the solution of the governing differential equations of a uniform homogeneous Timoshenko beam segment to interpolate the transverse displacement and rotation, is formulated in the context of the co-rotational approach. Using the formulated element, the large deflection response of a cantilever FGM beam with different taper cases is computed with the aid of the arc-length control algorithm in combination with the iterative Newton–Raphson method. Numerical examples are presented to show the accuracy and efficiency of the proposed element. The influence of the material distribution, taper ratio as well as the aspect ratio on the large deflections of the beams is numerically investigated in detail.

2. Tapered FGM beams

Consider a tapered FGM beam with length of L. Three following taper cases are considered in the present work

Case A: $A = A_0\left(1-\alpha\frac{x}{L}\right), \quad I = I_0\left(1-\alpha\frac{x}{L}\right)$
Case B: $A = A_0\left(1-\alpha\frac{x}{L}\right), \quad I = I_0\left(1-\alpha\frac{x}{L}\right)^3$
Case C: $A = A_0\left(1-\alpha\frac{x}{L}\right)^2, \quad I = I_0\left(1-\alpha\frac{x}{L}\right)^4$

where A_0, I_0 denote the area and moment of inertia of the section at $x = 0$, respectively; $0 \leqslant \alpha < 1$ is the taper ratio. Fig. 1 shows the geometry of the three considered cases of the beam in a co-ordinate system, (x, y, z), where the z axis directs along the thickness direction of the beam.

The beam is assumed to be formed from two different materials with volume fractions to be varied according to a power law as

$$V_1 = \left(\frac{z}{h}+\frac{1}{2}\right)^k, \quad V_2 = 1 - V_1, \tag{1}$$

where the subscript 1 and 2 indicate the material 1 and material 2 – the constituents of the FGM, respectively; k is the non-negative power law exponent, dictating the material variation through the beam thickness; h is the section height, which longitudinally varies for the case B and case C. According to the rule of mixture, the effective Young's modulus E, and the effective shear modulus G of the FGM beam are given by