Applied Mathematical Modelling 37 (2013) 1607–1617

Contents lists available at SciVerse ScienceDirect

Applied Mathematical Modelling

journal homepage: www.elsevier.com/locate/apm

Trigonometric wavelet-based method for elastic thin plate analysis

Wen-Yu He, Wei-Xin Ren *

Department of Civil Engineering, Hefei University of Technology, Hefei, Anhui Province 230009, China

ARTICLE INFO

Article history:
Received 19 September 2011
Received in revised form 14 April 2012
Accepted 21 April 2012
Available online 28 April 2012

Keywords:
Trigonometric wavelet
Thin plate
Elastic foundation
Formulation
Hierarchical method
Multi-resolution

ABSTRACT

Dealing with the boundary conditions is one of the difficult problems when using wavelet function as trial function to carry out structural analysis. In this paper, the two-dimensional tensor product trigonometric Hermite wavelet that has both good approximation characteristics of trigonometric function and multi-resolution, local characteristics of wavelet is proposed as trial function, and the united formulation of elastic bending, vibration and buckling of rectangle thin plate (on elastic foundation) with different boundary conditions is derived based on the principle of minimum potential energy. Two approaches, hierarchical and multi-resolution approach, are presented to improve calculation accuracy. The impact of proposed method is discussed by different numerical examples. Due to the Hermite interpolation properties, the proposed trigonometric wavelet method can process all kinds of boundary conditions conveniently. The solution accuracy of hierarchical method can be increased steadily with raising the order of wavelet, while the solution accuracy of multi-resolution method can be improved along with increasing the scale of wavelet.

1. Introduction

The elastic thin plates are widely used in engineering. It is of importance to carry out their bending, vibration and stability analysis and calculation. Taking wavelet function as interpolation function or trial function to analyze thin plates is a new method. Chen et al. solved the membrane structural vibration problem by using the elements constructed by spline wavelet, and the lifting algorithm taking advantage of the "two-scale relation" of wavelets was proposed [1]. Zhou et al. presented a modified Daubechies wavelet approximation for deflections of beams and square thin plates, in which boundary rotational degrees of freedom are included as independent wavelet coefficients [2]. Based on the modified approximations and Hamilton's principle, variation equations for dynamic, static and buckling problems of square plates are established, without requiring the wavelet approximations or the wavelet basis to satisfy any specific boundary condition in advance.

Han et al. developed a multivariable wavelet-based finite element formulation and solved the bending problems of thick plates based on the Hellinger–Reissner generalized variational principle with two kinds of independent variables [3]. Furthermore, wavelet-based plate element with a high precision and fast convergence by selecting appropriate spline wavelet scaling functions as the shape functions was proposed by the same authors [4,5]. Through introducing a transformation matrix that transforms the element deflection field represented by the coefficients of wavelets expansions from wavelet space to physical space, Xiang et al. constructed the wavelet Mindlin plate and thin plate elements based on B-spline wavelet on the interval [6,7]. Chen et al. developed a multi-resolution finite element method based on the interpolating wavelet transform and lifting scheme of the second generation wavelet [8]. Liu et al. established the solution formulations based on total Lagrangian approach for two-dimensional large deformation problems using the Daubechies wavelet [9].

* Corresponding author. Tel.: +86 731 82654349; fax: +86 731 85571736.
E-mail address: renwx@csu.edu.cn (W.-X. Ren).

0307-904X/$ - see front matter
http://dx.doi.org/10.1016/j.apm.2012.04.030

The advantages of wavelets are the multi-resolution, localization properties and various basis functions that are desirable for the solutions of various field problems. As the wavelet coefficients have no definite physical meaning, however, it is hard to deal with the boundary conditions when taking it as trail function. Dealing with the boundary conditions is one of the difficult problems when using wavelet function as trial function to carry out structural analysis. Up to now, there are two main approaches to handle this problem as follows:

(1) Selecting appropriate wavelet bases or linear combinations of scaling functions that satisfy the specified boundary conditions, then the corresponding system equations are formulated based on the variational principles [4–7]. Nevertheless, such formulations are only effective to the specified boundary conditions. The complicated or generalized boundary conditions of plates are difficult to meet.
(2) The system equations are first formulated by the variational principles, and then the formulations of definite boundary conditions that the displacement solutions should be satisfied are complemented [9]. However, the algebraic equations obtained by this method may be ill-posed.

Trigonometric series are often used to serve as the trail function in the weighted residual method in the field of field problem analysis [10], and turns out to be effective in approximating. Quak [11] presented the one-dimensional trigonometric wavelets for Hermite interpolation to take a full advantage of both trigonometric series and wavelets. The trigonometric wavelet was thereafter used to solve the field problems [12,13].

In this paper, the two-dimensional tensor product trigonometric Hermite wavelet that has both good approximation characteristics of trigonometric function and multi -resolution, local characteristics of wavelet is taken as a trial function. The unified formulation of elastic bending, vibration and buckling problems of rectangle thin plate (on elastic foundation) under different boundary conditions is derived accordingly based on the principle of potential energy minimum. Due to the Hermite interpolatory properties of trigonometric wavelets, all kinds of boundary conditions can be processed conveniently. Furthermore, the hierarchical and multi-resolution approaches, are proposed to improve calculation accuracy. They provide an alternatively way to improve the solution precision. A several numerical examples show that the presented method performs well in solving the bending, vibration and stability of rectangle thin plate (on elastic foundation), especially for natural vibration analysis. Compared to the conventional Rayleigh–Ritz method, the trigonometric wavelet method presented in this paper has several advantages in solving thin plate structures. It is demonstrated that the solution accuracy of hierarchical method can be increased steadily with raising the order of wavelet, while the solution accuracy of multi-resolution method can be improved along with increasing the scale of wavelet.

2. Trigonometric Hermite wavelet

For any $j \in N$, the scale functions of trigonometric Hermite wavelet [11] are defined by

$$\varphi_{j,0}^{0}(x) = \begin{cases} \frac{1}{2^{2j+2}} \frac{\sin^2(2^j x)}{\sin^2\left(\frac{x}{2}\right)} & x \notin 2\pi Z \\ 1 & x \in 2\pi Z \end{cases} \tag{1}$$

$$\varphi_{j,0}^{1}(x) = \begin{cases} \frac{1}{2^{2j+2}} \left(1 - \cos(2^{j+1}x)\right) \cot\left(\frac{x}{2}\right) & x \notin 2\pi Z \\ 1 & x \in 2\pi Z \end{cases} \tag{2}$$

The corresponding wavelet functions are as follows

$$\psi_{j,0}^{0}(x) = \frac{1}{3 \cdot 2^{2j+1}} \sum_{l=2^{j+1}+1}^{2^{j+1}-1} (3 \cdot 2^{2j+1} - l) \cos(lx) + \frac{1}{2^{j+1}} \cos(2^{j+1}x) \tag{3}$$

$$\psi_{j,0}^{1}(x) = \frac{1}{3 \cdot 2^{2j+1}} \sum_{l=2^{j+1}+1}^{2^{j+1}-1} \sin(lx) + \frac{1}{2^{2j+3}} \cos(2^{j+2}x) \tag{4}$$

The nodes for the interpolation processes are equally spaced on the interval $[0,2\pi)$ with a dyadic step size, i.e., $x_{j,n} = n\pi/2^j,\ j \in N_0,\ n = 0,1,2\cdots 2^{j+1}-1,\ \varphi_{j,n}^{0}(x) := \varphi_{j,n}^{0}(x - x_{j,n}),\ \varphi_{j,n}^{1}(x) := \varphi_{j,n}^{1}(x - x_{j,n})$.

The following interpolation properties hold for each k, $n = 0,1,2\ldots 2^{j+1}-1$:

$$\varphi_{j,n}^{0}(x_{j,k}) = \delta_{k,n}, \quad \varphi_{j,n}^{0\prime}(x_{j,k}) = 0, \quad \varphi_{j,n}^{1}(x_{j,k}) = 0, \quad \varphi_{j,n}^{1\prime}(x_{j,k}) = \delta_{k,n} \tag{5}$$

$$\psi_{j,n}^{0}(x_{j,k}) = \delta_{k,n}, \quad \psi_{j,n}^{0\prime}(x_{j,k}) = 0, \quad \psi_{j,n}^{1}(x_{j,k}) = 0, \quad \psi_{j,n}^{1\prime}(x_{j,k}) = \delta_{k,n} \tag{6}$$

$$\delta_{k,n} = \begin{cases} 1, & \mathrm{k} = \mathrm{n} \\ 0, & \mathrm{k} \neq \mathrm{n}. \end{cases} \tag{7}$$