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Unsteady MHD two-phase Couette flow of fluid–particle suspension

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ABSTRACT

The problem of two-phase unsteady MHD Couette flow between two parallel infinite plates has been studied taking the viscosity effect of the two phases into consideration. Unified closed form expressions are obtained for the velocities and the skin frictions for both cases of the applied magnetic field being fixed to either the fluid or the moving plate. The novelty of this study is that we have obtained the solution of the unsteady flow using the Laplace transform technique, D'Alemberts method and the Riemann-sum approximation method. The solution obtained is validated by assenting comparisons with the closed form solutions obtained for the steady states which have been derived separately and also by the implicit finite difference method. Graphical result for the velocity of both phases based on the semi-analytical solutions are presented and discussed. A parametric study of some of the physical parameters involved in the problem is conducted. The skin friction for both the fluid and the particle phases decreases with time on both plates until a steady state is reached, it is also observed to decrease with increase in the particle viscosity on the moving plate while an opposite behaviour has been noticed on the stationary plate.

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1. Introduction

The study of two-phase flow of fluid–particle is important in petroleum transport, wastewater treatment, combustion, power plant piping, corrosive particles in engine oil flow, smoke emission from vehicles and formation of raindrops. Its relevance is also seen in the field of mining, agriculture and food technologies. The flow of a dusty and electrically conducting fluid through a channel in the presence of a transverse magnetic field is also encountered in a variety of applications such as magnetohydrodynamic (MHD) generators, pumps, accelerators, and flow meters. In these devices, the solid particles in the form of ash or soot are suspended in the conducting fluid as a result of the corrosion and wear activities and the combustion processes in MHD generators and plasma MHD accelerators. In order to understand the presence of particles on the flow, there is the need to formulate the equation of motion for a two-phase fluid/particle system. To this end, Saffman [1] formulated the equations of motion of dusty gas by making some assumptions on dust particles. In his model, he neglected the interaction between the individual dust particles and also assumed that in the continuum hypothesis, the cloud of dust particles form a pseudo-fluid. Following Saffman [1], Nag et al. [2] studied the Couette flow of a dusty gas between two parallel infinite plates for both the impulsive and the accelerated start of one of the plates. The effect of particulate diffusion on the compressible boundary-layer flow of a two-phase suspension over a horizontal surface was examined by Chamkha [3]. Makinde and Chinyoka [4] investigated the unsteady flow and heat transfer of a dusty fluid between two parallel plates with variable viscosity and electric conductivity. The unsteady MHD Couette flow and heat transfer of a dusty conducting fluid between two parallel plates with variable viscosity was investigated by Attia [5]. Several other researchers; Gireesha et al. [6], Rashmi et al. [7] and Ghosh and Ghosh [8] studied the flow of dusty fluids under varying condition by excluding

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Nomenclature

B_0	constant uniform magnetic field
$f_1,\ f_2$	substance quantity of mixture in unit volume ($f_1 + f_2 = 1$)
$\mathcal{H}a$	Hartmann number
$\mathcal{K}'$	interaction coefficient
$\mathcal{K}$	dimensionless interaction coefficient
P	pressure gradient
s	Laplace parameter
t	dimensional time
η	dimensionless time
U	constant velocity
$V_f,\ V_p$	phase velocity component
$u'_f,\ u'_p$	velocity of fluid and particle
$u_f,\ u_p$	dimensionless velocity
z'	coordinate normal to channel
z	dimensionless coordinate
$v_f,\ v_p$	kinematic viscosity
$\rho_f,\ \rho_p$	density
σ	electrical conductivity
$\tau_f,\ \tau_p$	skin friction

the particle-phase viscous effects. Rahmatulin [9] developed a model which examines the unstable movement of the interface between the fluid and the particles and thereby included the particle-phase viscosity term in his model. This model has been improved by Fayzullyev [10], Latipov [11] and Drew [12] among other researchers under different conditions. Previous published work which use the concept of the particle-phase viscosity includes Gidaspow [13], Gadiraju et al. [14] and Chamkha and Ramadan [15]. Chamkha [16] studied the unsteady laminar flow and heat transfer of a particulate suspension in an electrically conducting fluid through channels and circular pipes in the presence of uniform transverse magnetic field which is formulated using a two-phase continuum model. He assumed the particle-phase viscosity to be constant in order to obtain a closed form solutions. Recebli and Kurt [17] examines the effect of the electrical and magnetic field applied perpendicular to the flow and each other on the two-phase steady flow in a gas pipe. By using the Reverse Laplace transformation approach, the coordinate based expressions for the unsteady state of phases was found. But analytical expressions of local velocities depending on time and coordinate were not obtained and therefore there were no solution for the unsteady state.

This work is aimed at providing the semi-analytical solution to the unsteady MHD two-phase Couette flow between two infinite parallel plate by considering the effect of the particle-phase viscous stresses. To the best of the authors' knowledge, this is yet to be addressed in open literature. The flow in the channel is generated by the impulsive movement of the lower plate. A semi analytical solution have been obtained for the velocities of both the fluid and the particle phases using the Laplace transform technique and the Riemann-sum approximation method, which has been used to obtain the skin frictions. The effect of the various dimensionless parameters entering into the problem are discussed. The semi-analytical solution presented in this article expand the database of known analytical/semi-analytical solutions for MHD two-phase flow which is important for validating numerical codes.

2. Mathematical analysis

The motion of an unsteady, laminar, viscous fluid/particle suspension between two infinite horizontal plates located at the $z' = 0$ and $z' = h$ planes is considered. The fluid phase is assumed to be electrically conducting, while the channel walls are assumed to be electrically non-conducting. A uniform transverse magnetic field is applied normal to the flow direction and it is assumed that no applied or polarization voltage exists. When $t > 0$ the lower plate ($z' = 0$) begins to move in its own plane with a velocity Ut^n where U is a constant and the upper plate ($z' = h$) remains fixed. The magnetic Reynolds number is assumed to be small so that the induced magnetic field and the Hall effects of MHD are negligible. It is assumed that the fluid and the particles are interacting as a continuum. Following Rahmatulin [9] model, the equation of motion with electromagnetic force added is given as shown below

$$\rho_f \frac{\partial \vec{V}_f}{\partial t} = -f_f P + f_f \mu_f \nabla^2 \vec{V}_f + K'(\vec{V}_p - \vec{V}_f) + \vec{J} \times \vec{B}; \quad \nabla \cdot \vec{V}_f = 0, \tag{1}$$

$$\rho_p \frac{\partial \vec{V}_p}{\partial t} = -f_p P + f_p \mu_2 \nabla^2 \vec{V}_p + K'(\vec{V}_f - \vec{V}_p); \quad \nabla \cdot \vec{V}_p = 0, \tag{2}$$

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