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On the role of radiative heat transfer in air gaps in vertical continuous casting

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ABSTRACT

A recent asymptotic thermomechanical model for the formation and evolution of air gaps in vertical continuous casting in round moulds is used as the basis for understanding the importance of radiative heat transfer between the mould wall and the solidified shell. Asymptotic analysis is used to systematically reduce the model to a moving boundary problem with a boundary condition in integro-differential form. In addition, a dimensionless parameter is found which determines whether radiation is significant or not for prescribed process operating conditions. Sample computations are carried out using parameters relevant to the continuous casting of copper, and the results are used to interpret earlier findings in the literature on the interaction between air-gap width and thermal radiation.

1. Introduction

Air-gap formation in the industrial continuous casting of metals and metal alloys has long been recognised as having an adverse effect on process efficiency. A schematic of the situation is given in Fig. 1, which shows molten metal, typically copper, aluminium or steel alloys, passing vertically downwards through a cooled mould, solidifying and being withdrawn at casting speed, V_{cast}. At a mould wall, there is typically first a region where liquid metal is in contact with the mould wall, followed by a region where the solidified shell is in contact. After this, at $z = z_{gap}$, an air gap begins to form between the solidified shell and the mould wall. Eventually, at some location $z = z_{mid}$, complete solidification occurs at the centreline. In particular, the formation of the air gap prohibits effective heat transfer between the mould and shell, leading to longer solidification lengths and requiring supplementary process design considerations, such as mould tapering. Observe also that, as we are principally interested in air-gap formation, we do not address the issue of breakout, in which the liquid metal has not totally solidified before the end of the mould; however, for the casting of copper, which is what is primarily being considered here, this is typically not a problem [1,2].

In view of the detrimental effect that the air gap has on process efficiency, mathematical models of varying degrees of complexity have been derived to describe the phenomenon. Early models for predicting the onset of air-gap formation were analytical [3–6]; most subsequent models have been solely numerical [7–13]. However, whilst able to capture the thermomechanical interaction of gap formation and evolution, such models are computationally expensive and unwieldy: for example, they do not give a qualitative understanding of the air-gap's dependence on different operating parameters, or indeed whether it is possible to avoid air-gap formation completely. An exception to all of the above is a sequence of recent models that use asymptotic methods [14–16]. However, whilst these models have put the thermomechanical coupling inherent in the problem on a sound mathematical footing, there is a now a need to consider more of the physics that is thought to be of relevance in continuous casting. To this end, the purpose of this paper is to consider the importance of radiative heat transfer

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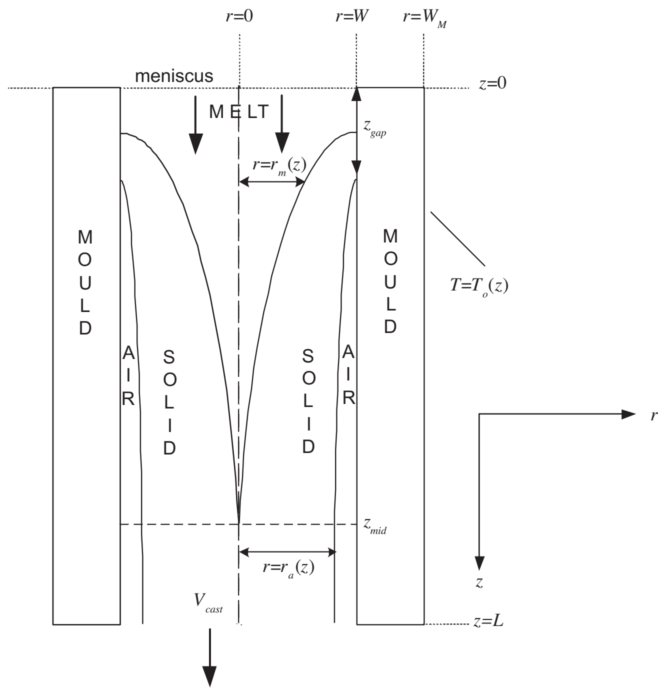

Fig. 1. Schematic of vertical continuous casting with air gap formation.

between the mould wall and the solidified shell, once the air gap has formed. As a framework for this study, we consider one of the thermomechanical models developed for vertical continuous casting in round moulds by Vynnycky [16]; this is a geometry that is of actual industrial relevance, and continues to form the basis of numerous recent casting studies [17–21].

Whether radiative heat transfer is in fact of importance in the air gap has been the subject of lengthy debate. It was initially anticipated by Mackenzie and Donald [22] that the governing mechanism for heat transport across the gap was radiation, a finding supported by Jacobi [23]; later, Ho and Pehlke [24] showed that conduction and radiation both played a role in heat transfer, whereas Nishida et al. [25] concluded that heat was mainly transferred by conduction. Thermal radiation has been explicitly included in some of the computationally intensive numerical models cited above [8,12], but was found to have little effect on the results obtained in others [26]. An alternative approach has been simply to use an inverse methodology which does not distinguish between conduction and radiation, but lumps these effects together in terms of a heat transfer coefficient for the gap [27–29]. Thus, although the generally accepted view is that radiation is of importance, its exact role remains unclear: for example, whether there are situations when it can be neglected, and the quantitative effect that it would have on shell solidification and air-gap evolution. Furthermore, whilst the works cited above have been for different types of casting processes, varying in geometry, operating temperature and cast material, it would clearly be useful to be able to identify a criterion that indicates whether or not thermal radiation will be important for a given process under prescribed operating conditions; this will be a by-product of the analysis presented in this paper.

The layout of the paper is as follows. In Section 2, we formulate a thermomechanical model that takes into account air-gap formation and thermal radiation for a continuous casting process in a round mould. In Section 3, we nondimensionalize the governing equations; asymptotic analysis then leads to a reduced model. In Section 4, a finite-element method is used to solve the resulting moving boundary problem. Conclusions are drawn in Section 5.

2. Mathematical formulation

We consider a steady state problem with cylindrical symmetry, as shown in Fig. 1, in which pure liquid metal at its melting temperature, T_{melt}, enters a mould region at $z = 0$, solidifies and is withdrawn at a casting speed V_{cast}; the extension to the case where the liquid metal is at a temperature greater than its melting temperature has been considered elsewhere, albeit for a different geometry [15], but the working assumption that we use allows us to take $r_{\mathrm{m}}(0) = W$ and thereby to avoid extraneous details that are not essential here. Subsequently, an air gap starts to form at the inner mould surface at $z = z_{\mathrm{gap}}$. For $0 < z < z_{\mathrm{gap}}$, solidification occurs in the region $r_{\mathrm{m}}(z) < r < W$, whereas for $z > z_{\mathrm{gap}}$, air occupies $r_{\mathrm{a}}(z) < r < W$ and solid occupies $r_{\mathrm{m}}(z) < r < r_{\mathrm{a}}(z)$. Eventually, after complete solidification has occurred at $z = z_{\mathrm{mid}}$, the solid region occupies $0 < r < r_{\mathrm{a}}(z)$.