Applied Mathematical Modelling 37 (2013) 2469–2483

Contents lists available at SciVerse ScienceDirect

Applied Mathematical Modelling

journal homepage: www.elsevier.com/locate/apm

Control of a novel class of fractional-order chaotic systems via adaptive sliding mode control approach ☆

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ARTICLE INFO

Article history:
Received 25 September 2011
Received in revised form 10 April 2012
Accepted 4 June 2012
Available online 12 June 2012

Keywords:
Fractional-order chaotic system
Adaptive sliding mode control
Chaos
Uncertainty
External disturbance

ABSTRACT

In this paper, an adaptive sliding mode controller for a novel class of fractional-order chaotic systems with uncertainty and external disturbance is proposed to realize chaos control. The bounds of the uncertainty and external disturbance are assumed to be unknown. Appropriate adaptive laws are designed to tackle the uncertainty and external disturbance. In the adaptive sliding mode control (ASMC) strategy, fractional-order derivative is introduced to obtain a novel sliding surface. The adaptive sliding mode controller is shown to guarantee asymptotical stability of the considered fractional-order chaotic systems in the presence of uncertainty and external disturbance. Some numerical simulations demonstrate the effectiveness of the proposed ASMC scheme.

1. Introduction

In the past few decades, much attention has been drawn for the study of fractional calculus [1–7]. As a branch of mathematical analysis, fractional calculus is a generalization of integration and differentiation to arbitrary non-integer orders. The applications of fractional calculus have been intensively investigated in many research fields, covering automatic control, informatics, materials, physics and etc. Especially, the study on the dynamics of fractional-order differential systems has attracted increasing affection from many researchers [8–13].

Chaos as a very interesting nonlinear phenomenon has been intensively investigated in many fields of science and technology [14–17] over the last decades. It has been also demonstrated that some fractional-order differential systems [18–21] behave chaotically or hyper-chaotically, such as the fractional-order Chen system [22,23], the fractional-order financial system [24,25]. Recently, the fractional-order chaotic systems have become a hot topic. In particular, control and synchronization [26–29] of the fractional-order chaotic systems have attracted attention from various scientific fields.

The main feature of SMC [23–25,30–32] is to switch the control law to force the states of the system from the initial states onto some predefined sliding surface. The system on the sliding surface has desirable properties such as stability and disturbance rejection capability. SMC is well known as an effective robust control strategy. Yin et al. [25] presented the SMC law for the chaos control of a class of fractional-order nonlinear systems. In addition, adaptive control [33] can overcome uncertainties

☆ This work was supported by National Basic Research Program of China (2010CB732501) and the National Natural Science Foundation of China (NSFC-60873102).

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http://dx.doi.org/10.1016/j.apm.2012.06.002

and external disturbances. So the ASMC has the advantage of combining the robustness of the sliding mode control with the tracking abilities of the adaptive control. However, there are few related results reported on the ASMC of fractional-order chaotic systems.

In this paper, we first introduce a novel class of fractional-order chaotic systems. Then, an ASMC law is proposed to control chaos in such fractional-order systems. The controller is developed to stabilize the novel fractional-order chaotic systems, even if the fractional-order systems with uncertainty and external disturbance. Numerical simulations show that the proposed method can easily eliminate chaos and stabilize the system on the sliding surface.

The paper is presented as follows: in Section 2, basic definitions of fractional calculus, notations and numerical algorithms are given. In Section 3, the general description of fractional-order chaotic systems is presented. Section 4 proposes the employment of the sliding mode control method to control chaos in the systems. Numerical simulation results are shown in Section 5. Finally, conclusion is addressed in section 6.

2. Basic definition and preliminaries

There exist many definitions of fractional derivative [34]. The most well-known definition is Riemann–Liouville (RL) fractional derivative, defined by

$$D_t^q f(t) = \frac{d^q f(t)}{dt^q} = \frac{1}{\Gamma(m-q)} \frac{d^m}{dt^m} \int_0^t \frac{f(\tau)}{(t-\tau)^{q+1-m}} d\tau, \tag{1}$$

where m is the first integer which is not less than q, i.e. $m-1 \leqslant q < m$ and $\Gamma(\cdot)$ is the well-known Euler's gamma function

$$\Gamma(s) = \int_0^\infty t^{s-1} e^{-t} dt. \tag{2}$$

The terminal value indicates the lower limit in the integral (1). It may be a nonzero value in the general definition of fractional derivative. The Laplace transform of the Riemann–Liouville fractional derivative is given by

$$L\left\{\frac{d^q f(t)}{dt^q}\right\} = v^q L\{f(t)\} - \sum_{k=0}^{m-1} v^k \frac{d^{q-1-k} f(0)}{dt^{q-1-k}}, \quad m-1 < q \leqslant m, \tag{3}$$

where L means Laplace transform and v is a complex variable. Note that in (3) the non-integer order derivative of the function at $t=0$ is required. This may be problematic in practical applications. So, the Caputo definition, sometimes called smooth fractional derivative, is preferred which is

$$D_t^q f(t) = \frac{d^q f(t)}{dt^q} = \begin{cases} \frac{1}{\Gamma(m-q)} \int_0^t \frac{f^{(m)}(\tau)}{(t-\tau)^{q+1-m}} d\tau, & m-1 < q < m, \\ \frac{d^m f(t)}{dt^m}, & q = m, \end{cases} \tag{4}$$

where m is the first integer which is not less than q. The Laplace transform of the Caputo fractional derivative is given by:

$$L\left\{\frac{d^q f(t)}{dt^q}\right\} = v^q L\{f(t)\} - \sum_{k=0}^{m-1} v^{q-1-k} f^{(k)}(0), \quad m-1 < q \leqslant m. \tag{5}$$

In (5), note that only integer orders derivatives of the function appear in the Laplace transform of the Caputo derivative. When the initial conditions are all zero, (5) reduces to

$$L\left\{\frac{d^q f(t)}{dt^q}\right\} = v^q L\{f(t)\}. \tag{6}$$

On the other hand, the initial conditions for the fractional differential equations (FDEs) with the Caputo derivative are in the same form as for integer-order derivatives which have clear physical meaning. So, the Caputo fractional derivative is more popular than the Riemann–Liouville fractional derivative, when modeling real-world phenomena with FDEs. Hence, the Caputo derivative is used in this paper.

Lemma 2.1 (Barbalat's lemma [35]). *If $\eta : R \to R$ is a uniformly continuous function for $t \geqslant 0$ and if the limit of the integral $\int_0^t \eta(\omega) d\omega$ exists and is finite, then $\lim_{t\to\infty} \eta(t) = 0$.*

Lemma 2.2. *The following equality is valid for every positive scalar α and given scalar β.*

$$\beta \tanh(\alpha\beta) = |\beta \tanh(\alpha\beta)| = |\beta||\tanh(\alpha\beta)| \geqslant 0. \tag{7}$$

Proof. From the definition of $\tanh(\alpha) = \frac{e^\alpha - e^{-\alpha}}{e^\alpha + e^{-\alpha}}$, the following equality can be obtained