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Axi symmetric 2D simulation and numerical heat transfer characteristics for the calibration furnace in a rectangular enclosure

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ABSTRACT

This paper presents axi symmetric 2D numerical investigation of the spherical thermocouple calibration furnace in a rectangular enclosure. The focus is on the flow structure inside the Saturn (a hollow spherical cavity), external flow behavior due to annulus block heating and the surface temperature uniformity. Mesh sensitivity analysis is adopted to extract the mesh with minimum number of nodes but with fast convergent finite element solution. The basic strategy here is that temperature perturbation error at a single point instead of a single element contributed to the total perturbation error qualitatively remains the same. Agreement between numerical simulation results and the experiment results is good with a maximum temperature deviation 10 °C for the cavity temperature 400 °C. Finally, standard numerical temperature uncertainty due to variation in thermal conductivity is computed through the sensitivity coefficient using uncertainty analysis.

1. Introduction

Calibration of radiation thermometers [1] is one of the important research activities in the field of metrology. Calibration is an act of adjusting instrument by comparison against standard. Radiation thermometers are calibrated using a standard surface of known temperature. The Saturn surface is one of such standards used in the calibration. The Saturn is a hollow spherical cavity embedded in the calibration furnace. The important advantage for choosing the spherical cavity is that the cavity behaves like a black body (emissivity ϵ is almost 1) despite the fact that it is a grey body. Hence, more accuracy is achieved in the emitted radiation measured by radiation thermometer. Radiation thermometers are sighted at the inner surface of the spherical cavity through the furnace aperture (see Fig. 1).

Calibration furnace (see Fig. 1) consists of six concentric layers and each layer is embedded in the other. The first outermost layer is in steel having a thickness of 0.02 m. Inside this layer, a ceramic fibre is established over the entire annulus of diameter 0.051 m to avoid heat losses. Ceramic fibre envelopes the annulus block heating which consists of helicoidal copper coil with best thermal properties arranged internally to the cement block. Copper coil is formed like a serpentine spread through all zones interior to the block. Power supply is provided to the copper coil through the small gaps in layers. Inside this block heating, there is another ceramic fibre envelopes the spherical cavity to withstand elevated temperatures.

Nine calibrated positions marked as F1, F2, F3, F4, F5, C6, C7, C8 and C9 are selected for temperature measurement on the spherical cavity surface and they are sketched systematically in Fig. 2. F1 is marked exactly opposite to the frontal diameter of the spherical cavity. Positions F2, F3, F4 and F5 are indicated on the surface cross section so that they are on the same side (opposite to frontal view) and they are on the same circle. This cross sectional surface cut makes a solid angle of 30Sr at the

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Nomenclature

r, z	cylindrical coordinates
R, Z	dimensionless coordinates in r-, z-directions
U, V	dimensionless velocities in r-, z-directions
T_s	surface temperature (°C)
T_{amb}	ambient temperature (°C)
L	characteristic length (length of the layer or diameter of the sphere)
AR	aspect ratio (width to height ratio)
Ra	Rayleigh number $= \frac{g\beta\Delta TL^3}{\nu\alpha}$
Nu_{loc}	local Nusselt number $= \frac{hR}{k}$
k	thermal conductivity (W/m K)
Pr	Prandtl number (=0.71 at $T_{ref} = 20$ °C)
$\mathbf{n}$	unit normal vector
$\mathbf{E}$	number of elements
$\mathbf{N}$	number of nodes
h	maximum element size (length of largest edge in an element) (m)
$\mathtt{h}$	convective heat transfer coefficient (W/m^2 K)
Greeks	
ϕ	azimuthal angle
ρ	density of the air (kg/m^3)
α	thermal diffusivity (m^2/s)
β	coefficient of volumetric thermal expansion (/°C)
ϵ	emissivity of the cavity
θ	dimensionless temperature
σ	Stefan Boltzmann constant (5.67×10^{-8} W/m^2 K^4)

center of the spherical cavity. Positions C6, C7, C8 and C9 are selected on the great circle so that none of them comes at the frontal side. In an experiment, the measured surface temperature at nine thermocouple positions are not the same for a particular cavity temperature.

An important requirement for calibration of radiation thermometers is that the cavity must be isothermal ($\Delta T = 0, Q \neq 0$). But it is difficult to maintain isothermal character for the spherical cavity although uniform heating provided. This is the drawback of the experiment and consequently results might be inaccurate. Reason being (1) heat losses due to the aperture (2) heat losses due to air gaps in the ceramic material. In order to understand thoroughly, it is essential to study behavior of temperature profile for particular aperture size of the spherical cavity. If the aperture size is large, heat radiation cannot produce sufficient number of reflections required to increase the cavity emissivity. Cavity emissivity against the surface temperature uniformity plot determines the heat dissipation rate through the aperture. The developed mathematical model and then numerical simulation gives better insight and understanding of the experiment, optimize uncertainties in the solution that suggest a methodology to improve quality of the equipment.

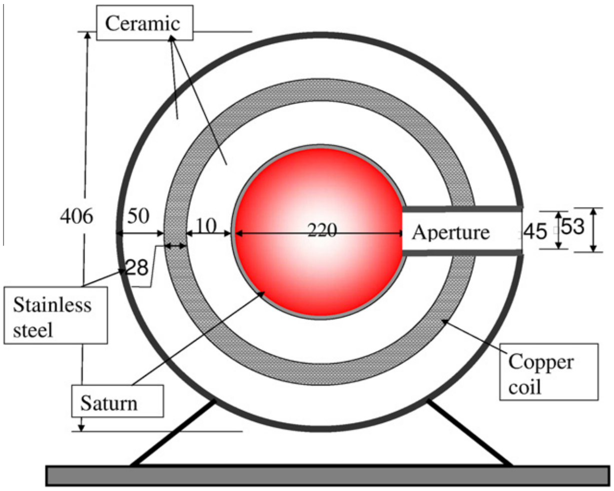

Fig. 1. Calibration furnace (dimensions expressed in mm).