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Forced vibration of curved beams on two-parameter elastic foundation

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ABSTRACT

Forced vibration analysis of curved beams on two-parameter elastic foundation subjected to impulsive loads are investigated. The Timoshenko beam theory is adopted in the derivation of the governing equation. Ordinary differential equations in scalar form obtained in the Laplace domain are solved numerically using the complementary functions method. The solutions obtained are transformed to the real space using the Durbin's numerical inverse Laplace transform method. The static and forced vibration analysis of circular beams on elastic foundation are analyzed through various examples.

1. Introduction

Beams and plates resting on elastic foundations have wide application in engineering practice. The dynamic analysis of beams is investigated using various foundation models. Numerous studies have been performed to investigate the static deflection and dynamic response of the beams resting on various elastic foundations.

There are few studies on the static analysis of curved beams on elastic foundation. Kıral and Ertepınar [1] investigated the isothermal behavior of planar rods resting on an elastic foundation and subjected to a static loading. Dasgupta and Sengupta [2] presented a new formulation based on curved three noded isoparametric beam elements with or without an elastic base throughout its length. Banan et al. [3] developed a general finite element formulation for spatial curved beams and arches on elastic foundation. Haktanır and Kıral [4] studied the behavior of continuous and elastically supported helicoidal structures by the stiffness matrix approach based on transfer matrix method. Aköz and Kadıoğlu [5] analyzed circular beams with variable cross-sections on elastic foundation under arbitrary loading by finite element method.

Similarly, there are also few studies on the free vibration analysis of curved beams on elastic foundation. Wang and Brannen [6] studied natural frequencies of curved beams on elastic foundation. They showed the effects of the opening angle of the curved beam and foundation constants on the natural frequencies of the beam. Issa [7] and Issa et al. [8] examined natural frequencies of curved beams on Winkler and Pasternak foundation. They obtained the dynamic stiffness matrix of a curved member of constant section. Wu and Parker [9] obtained free vibration of a thin ring a general elastic foundation. They determined natural frequencies and vibration modes having closed-form expressions for a ring with a circumferentially varying foundation of very general description. Çalım and Akkurt [10] studied static and free vibration analysis of straight and circular beam on elastic foundation.

However, there is only one study on the forced vibration analysis of circular rings on a tensionless foundation by Celep [11]. He studied the problem of a thin elastic circular ring on tensionless Winkler foundation. He derived the governing equations of the problem by employing Lagrange's equations for solving static and in-plane vibration of an elastic ring supported on an elastic Winkler foundation.

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There exist many papers in literature related to beams and plates on elastic foundations. The researchers, in general, have been interested in the static, dynamic, and stability analysis of the beams and plates on elastic foundations. Although, the static and dynamic analysis of straight beams and plates on elastic foundations are also commonly seen, there relatively few studies exist in literature examining the free and force vibrations of curved beams on elastic foundation as given in previous paragraphs.

In this study, an efficient method is introduced for the analysis of the forced vibration of curved beams on two-parameter elastic foundation under impulsive loads in the Laplace domain. In this method, the governing equations for naturally twisted and curved spatial rods obtained using the Timoshenko beam theory are rewritten for curved beams. The curvature of the rod axis, effect of rotary inertia and, shear and axial deformations are considered in the formulation. The element dynamic stiffness matrix is calculated in the Laplace domain by applying the complementary functions method to the differential equations in canonical form [12–18]. This provides great convenience in the solution of the problems having general boundary conditions as the desired accuracy is obtained by taking only a few elements as opposed to high number of elements needed in finite element analysis. Moreover, ordinary differential equations with variable coefficients can also be solved exactly in Laplace domain by using the complementary functions method. The solutions obtained in the Laplace domain are then transformed to the time domain using the Durbin's numerical inverse Laplace transform method [12–22].

2. The governing equations

Consider a naturally curved and twisted spatial slender rod. The trajectory of geometric center G of the rod is defined as the rod axis and its position vector at $t = 0$ is given by $\mathbf{r}^o = \mathbf{r}^o(s, 0)$ where s is measured from an arbitrary reference point $s = 0$ on the axis (Fig. 1). A moving reference frame is defined by the unit vectors **t**, **n**, **b** with the origin on the rod axis, where **t**, **n** and **b** are tangent, normal and binormal vectors, respectively. The following differential relations among the unit vectors **t**, **n**, **b** can be obtained with the aid of the Frenet formulae [23]:

$$\partial\mathbf{t}/\partial s = \chi\mathbf{n}, \quad \partial\mathbf{n}/\partial s = \tau\mathbf{b} - \chi\mathbf{t}, \quad \partial\mathbf{b}/\partial s = -\tau\mathbf{n}, \tag{1}$$

where χ and τ are the curvature and the natural twist of the axis, respectively. For planar rods $\tau = 0$, and for straight rods $\chi = \tau = 0$.

Let the displacement of a point on the rod axis be $\mathbf{U}^o(s, t)$, and the rotation of the cross-section about an axis passing through the geometric center G be $\mathbf{\Omega}^o(s, t)$. Assuming the displacements and the deformations are infinitesimal, and that the material of the rod is homogenous, linear elastic and isotropic the governing equations of a space rod are obtained in vectorial form as:

$$\frac{\partial\mathbf{U}^o}{\partial s} + \mathbf{t}\times\mathbf{\Omega}^o - \mathbf{C}^{-1}\mathbf{T}^o = 0, \quad \frac{\partial\mathbf{\Omega}^o}{\partial s} - \mathbf{D}^{-1}\mathbf{M}^o = 0, \tag{2}$$

$$\frac{\partial\mathbf{T}^o}{\partial s} + \mathbf{p}^{(ex)} - \mathbf{p}^{(in)} = 0, \quad \frac{\partial\mathbf{M}^o}{\partial s} + \mathbf{t}\times\mathbf{T}^o + \mathbf{m}^{(ex)} - \mathbf{m}^{(in)} = 0, \tag{3}$$

where the inertia force vector is $\mathbf{T}^o$ the inertia moment vector is $\mathbf{M}^o$ and $\mathbf{p}^{(ex)}$ and $\mathbf{m}^{(ex)}$ are the external distributed load and external distributed moment vectors per unit length of axis, respectively. The mass density ρ, the inertia force $\mathbf{p}^{(in)}$ and the inertia moment $\mathbf{m}^{(in)}$, per unit length of the rod axis are given as

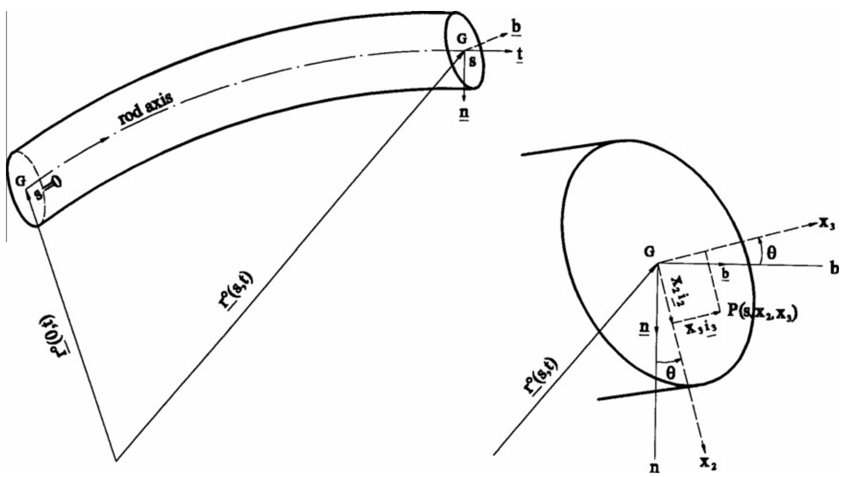

Fig. 1. The rod geometry.