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On a problem of nonstationary two-dimensional motion of micropolar fluid when normal stress and tangential velocity are given on the boundary

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ABSTRACT

The object of this paper is to investigate the solution of nonstationary motion of micropolar fluid in the half-plane when the normal stresses and tangential velocities are given on the boundary. The Laplace–Fourier transform technique is used to point out the solution by quadratures. Numerical results of the physical quantities such as tangential and normal velocities, pressure, microrotation, stresses and momentums are obtained and displayed graphically. The problem could be met in the study of the vibrations of a memberance or a plate contacting with the fluid.

1. Introduction

The theory of micropolar fluid was first proposed by Eringen [1]. Extensive review of the theory and applications which considered the boundary condition of either a constant wall temperature or constant heat flux can be found in the review article by Ariman et al. [2], Char and Chang [3], Pop et al. [4], Kim and Korea [5], Wang and Chen [6], Nazar et al. [7], Siddshwar and Sri Krisha [8] and Cheng [9]. The two dimensional parallel shear flow of a linear micropolar fluid is analyzed and compared with the colloid suspensions [10]. Solution of the fundamental Oseen problem for the 2-dimensional flow of micropolar fluid is obtained in explicit form under a certain restriction on the physical parameters of the problem [11]. Kamel [12] used a perturbation series to solve and analyzed the system of partial differential equations of the flow of a creeping polar fluid between eccentric rotating cylinders. The problem of the two-dimensional flow of a nonstationary micropolar fluid in the half plane for which the shear stresses are given on the boundary was solved by quadratures [13]. In [14], the constructed causal fundamental solutions for the slow 2D flow of a micropolar fluid requires the factorization of a fourth order P.d.O. into the quadratic operators. Dong [15] is concerned with the existence and regularity of the global attractors of 2D micropolar fluid flows in some unbounded domains. Lok et al. [16] investigated steady two-dimensional periodic motion of a micropolar fluid near an infinite array of moving walls. Chernous and El-Sirafy [17] solved the boundary value problem of slow two-dimensional nonstationary motion of viscous incompressible fluid in the half-plane for which the normal stresses and tangential velocities are given on the boundary.

In the present paper, an attempt is made to generalize the results of Chernous and El-Sirafy to the class of the micropolar fluids for the given normal stresses and tangential velocities on the boundary. The normal derivatives of microrotation vanish on the boundary. Also, we consider the initial velocities and the initial microrotation are zero. The Laplace–Fourier

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Nomenclature

$H(x,t)$	normal stress on the boundary
$H(t)$	Heaviside function
$K_o(z)$	Macdonald function of zero order of the argument z
m_{xx}, m_{yy}, m_{zz}, m_{xy}, m_{yx}, m_{xz}, m_{zx}, m_{yz}, m_{zy}	momentums
P	pressure
$\vec{q}$	velocity vector
t	time
$u(x,y,t)$, $\upsilon(x,y,t)$	tangential and normal components of the velocity
$U(x,t)$	normal velocity on the boundary
x and y	dimensional components,
$\widehat{xx}\,\widehat{yy}$, $\widehat{zz}$, $\widehat{yx}$, $\widehat{xy}$, $\widehat{yz}$, $\widehat{zy}$, $\widehat{xz}$, $\widehat{zx}$	stresses
α, β, γ, μ, k, J	viscosity coefficients
$\vec{v}$	microrotation vector
ρ	fluid density

transforms technique is used to point out the solution by quadratures. Numerical results of tangential and normal velocities, pressure, microrotation, stresses and momentums are obtained and displayed graphically, This problem could be met in the study of the vibrations of a membrance or plane contacting with the micropolar fluid.

2. Mathematical analysis

The linearized equations of motion of an incompressible micropolar fluid in the absence of both external forces and body couples [1] are

$$\left.\begin{aligned} &\nabla.\vec{q} = 0,\\ &-(\mu+k)\nabla\wedge(\nabla\wedge\vec{q}) + k\nabla\wedge\vec{v} - \nabla P = \frac{\partial\vec{q}}{\partial t},\\ &(\alpha+\beta+\gamma)\nabla(\nabla.\vec{v}) - \gamma\nabla\wedge(\nabla\wedge\vec{v}) + k\nabla\wedge\vec{q} - 2k\vec{v} = \rho J\frac{\partial\vec{v}}{\partial t}. \end{aligned}\right\} \tag{1}$$

We consider the case of two-dimensional unsteady motion of a micropolar incompressible fluid in the half-plane $(-\infty < x < \infty,\ y > 0|t > 0)$. In this case

$$\vec{q} = (u, \upsilon, 0), \quad \vec{v} = (0, 0, v).$$

The system (1) is reduced to

$$\mathrm{Re}\left\{\frac{\partial w}{\partial z}\right\} = 0, \tag{2a}$$

$$\rho\frac{\partial w}{\partial t} = (\mu+k)\nabla^2 w - 2\frac{\partial}{\partial\bar{z}}(P + ikv), \tag{2b}$$

$$\rho J\frac{\partial v}{\partial t} = \gamma\nabla^2 v - 2kv + 2kIm\left\{\frac{\partial w}{\partial z}\right\}, \tag{2c}$$

where $z = x + iy$ and $w = u + i\upsilon$.

Now we need to solve the system (2) subject to the boundary conditions

$$u(x,0,t) = U(x,t), \tag{3}$$

$$\left[P + (2\mu+k)\frac{\partial u}{\partial x}\right]_{y=0} = -H(x,t), \tag{4}$$

$$\left.\frac{\partial v}{\partial y}\right|_{y=0} = 0, \tag{5}$$

$$w(x,\infty,t) = v(x,\infty,t) = 0 \tag{6}$$