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Towards the analytic characterization of micro and nano surface features using the Biharmonic equation

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ABSTRACT

The prevalence of micromoulded components has steadily increased over recent years. The production of such components is extremely sensitive to a number of variables that may potentially lead to significant changes in the surface geometry, often regarded as a crucial determinant of the product's functionality and quality. So far, traditional large-scale quality assessment techniques have been used in micromoulding. However, these techniques are not entirely suitable for small scales. Techniques such as atomic force microscopy (AFM) or white light interferometry (WLI) have been used for obtaining full three-dimensional profiles of micromoulded components, producing large data sets that are very difficult to manage. This work presents a method of characterizing surface features of micro and nano scale based on the use of the Biharmonic equation as means of describing surface profiles whilst guaranteeing tangential (C^1) continuity. Thus, the problem of representing surface features of micromoulded components from massive point clouds is transformed into a boundary-value problem, reducing the amount of data required to describe any given surface feature. The boundary conditions needed for finding a particular solution to the Biharmonic equation are extracted from the data set and the coefficients associated with a suitable analytic solution are used to describe key design parameters or geometric properties of a surface feature. Moreover, the expressions found for describing key design parameters in terms of the analytic solution to the Biharmonic equation may lead to a more suitable quality assessment technique for micromoulding than the criteria currently used. In summary this technique provides a means for compressing point clouds representing surface features whilst providing an analytic description of such features. The work is applicable to many other instances where surface topography is in need of efficient representation.

1. Introduction

There are many materials for which the surface topography is a crucial characteristic. There are too many examples to list comprehensively, but a few instances serve to illustrate the range of materials and applications. One such is the work in [1] on surface geometry, wettability, roughness and other properties of composite polylactic acid and calcium phosphate glass scaffolds with a biological application. Another example whereby 3D shape characterization has been employed [2] involves a lunar soil simulant material with the aim of using the resulting shapes in a series of flow particle simulations. Some other examples include shape characterization of cement particles [3], characterization of ZnO [4], shape characterization of sol–gel zinc silicate glass particles [5] and large defects in alumina ceramics [6].

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The above examples illustrate the relevance of shape characterization of particles or surfaces of a given material. However, shape characterization is also useful to describe or quantify the overall properties of manufactured parts such as the ones obtained through micromoulding.

Micromoulding refers to the mass production process through which devices with micro and nano scale surface features are manufactured [7] (the overall size of the components analyzed in this work is in the order of microns with nanometric surface features). This has become steadily more important over the past 30 years, when it was forecasted as one of the main manufacturing technologies of the 21st century [8]. Its current relevance is mainly a byproduct of the constant development of portable electronic devices. This industry also manufactures devices for micro-optics, and life sciences applications. Most of the devices produced by this process are made of polymeric materials since this type of material offers the possibility of being tailored to the specific needs of a given application [8].

Micromoulding is affected by a number of variables such as the injection rate at which the moulding cavity is filled, injection and holding pressures, mould and melt temperature and cooling time [9]. Small variations in such variables may lead to a significant change in the shape and properties of the final product. Extensive research has been carried out on the influence that these variables have on the final shape of the end product. These include studies on flow visualization in micromoulding for determining cooling rates within the mould and research on the mechanical properties of micro components using atomic force microscopy [10–13]. Uncontrolled changes on the shape and properties of the component are clearly unwanted and quality control methods must be implemented. However, quality assessment techniques previously available, often applied to larger scale components, cannot offer the precision required to verify the quality of micromoulded components. Given that the product's functionality is directly related to its surface geometry, techniques that quantify the surface directly are desirable for the control of quality.

Research on product property measurement has been carried out and has been used to determine dimensional properties, consequently allowing for full three-dimensional assessments. Among the techniques readily available are:

- *White light interferometry (WLI).* This is a non-contact 3D surface measurement technique with an associated accuracy of fractions of a micron and is combined with an optical setup for visualizing structures at microscopic scales. It is set up in such a way that the height of a given two-dimensional point is determined by the relation between the path lengths of two different beams.
- *Atomic force microscopy (AFM).* The resolutions obtained with this technique are equivalent to fractions of a nanometer. The information is gathered via a probe with a very sharp nanometer sized tip and a sensitive piezoelectric actuator so that accurate small-scale scanning can be obtained. Its advantage over electron microscopy is that it provides a true three-dimensional profile, and that the instruments can perform in ambient conditions and even in liquids [14,13].
- *Nanoindenting.* This technique is so flexible that can be applied to different materials ranging from metals and plastics and in forms varying from thin films to powders [15]. A nanometer scale diamond tip is driven into the material surface with displacement and load monitored.
- *Extensive depth of field.* The traditional optics concept of extensive depth of field has been applied within micromoulding as a technique for obtaining three-dimensional profiles by recording a number of images at different depths transversally to the focus plane. Then, the set of images is processed so that a fully focused three-dimensional representation of the component is obtained [16].
- Utrasonic techniques, which combine the use of ultrasound and an optical laser to determine the dimensions of a given cavity [17].

In summary, surface measurement techniques employed for characterizing micro and nano scale surface features nowadays are generally associated with massive amounts of data in the form of three-dimensional point clouds. Management of these data is difficult in practice. Thus, a surface characterization technique capable of describing the surface geometry accurately whilst compressing the data set would be ideal.

This work proposes the use of partial differential equations (PDEs) as means of obtaining a surface representation of a given micromoulded component semi-automatically. PDEs have been previously used as a surface generation technique to produce parametric surfaces in other generally larger scale applications and computer-aided design [18] and recently

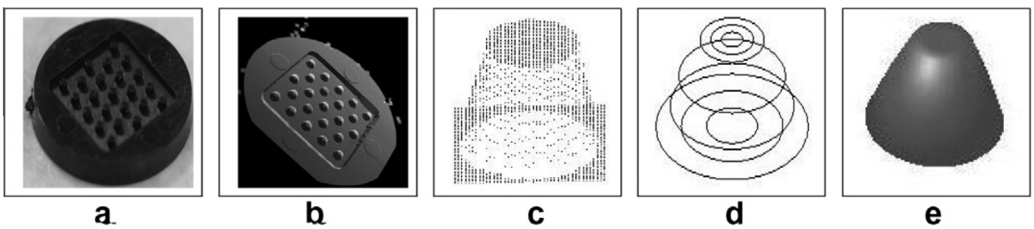

Fig. 1. Schematic representation of the surface characterization process proposed in this work. The stages illustrated here start at the production of micromoulded components (a) followed by the extraction of three-dimensional data (b). These data are analyzed so that they are divided into regions representing each surface feature (c). Then, a set of boundary curves representing the surface profile of the feature is obtained (d) and the process finishes with the generation of a PDE surface representation of each surface feature within a given component (e).