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Periodic solution of a chemostat model with variable yield and impulsive state feedback control ☆

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ABSTRACT

In this paper, a chemostat model with variable yield and impulsive state feedback control is considered. We obtain sufficient conditions of the globally asymptotical stability of the system without impulsive state feedback control. We also obtain that the system with impulsive state feedback control has periodic solution of order one. Sufficient conditions for existence and stability of periodic solution of order one are given. In some cases, it is possible that the system exists periodic solution of order two. Our results show that the control measure is effective and reliable.

1. Introduction

Recently, impulsive differential equations have aroused scholars' enormous interest. Meng [1,2], Jiao [3,4], Shi [5,6], Zhao [7,8], Liu [9,10], and Wei [11] have investigated and well studied the models with fixed instants of impulse effect. As for the models with impulsive state feedback control, Zeng [12], Jiang [13] and Tang [14] discussed prey–predator models and obtained the complete expression of the periodic solution. Since the models with impulsive state feedback control meet the need of biological control in practice, the study about the models has become scholars' focus. However, few papers have discussed the chemostat system using the impulsive differential equation with state feedback control.

The chemostat is a basic piece of laboratory apparatus, and it has begun to occupy an increasingly central role in ecological studies. As a tool in biotechnology, the chemostat plays an important role in bio-processing. Chemostat has been used widely to study bacterial metabolism, population genetics and plasmid stability, mostly because these are the simplest and most easily constructed types of continuous cultures. the mathematical models on the culture of the microorganisms have been investigated by many paper. For example, mathematical [15–20] and experimental [21–23] models exhibit the competitive exclusion principle only one species survives. Several modifications of the chemostat have been made to ensure the coexistence of species on a single nutrient [24,25,21,23]. However, there are a lot of factors such as temperature, dissolved oxygen content affecting the growth and reproduction of the microorganisms in the process of bio-reacts. In some cases, it is necessary to regulate the bio-react according to the concentration of microorganisms.

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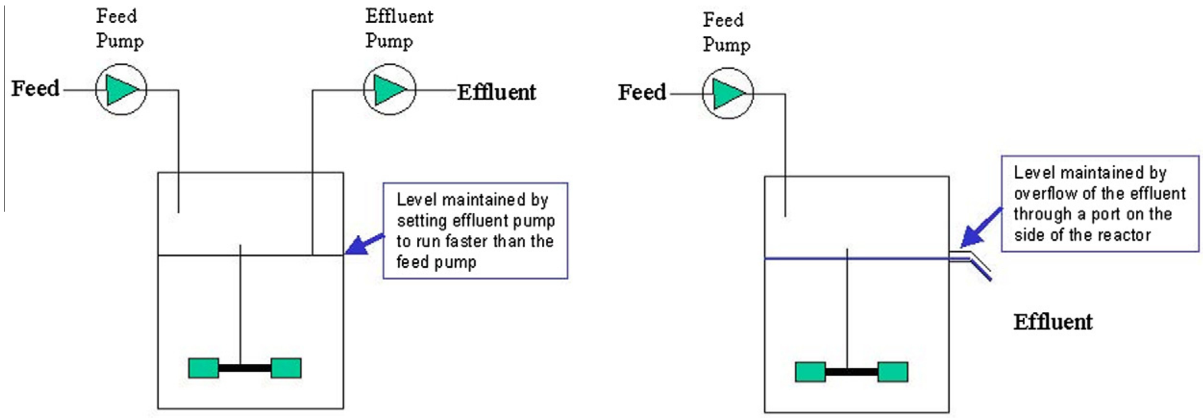

Fig. 1. Two type of chemostat.

By using a pump (seen on the left of Fig. 1) or an overflow system (seen on the right of Fig. 1), the volume of the chemostat can be controlled either. In some cases, the high concentration of the microorganisms will often have caused a series of adverse effects. To regulate the concentration of the microorganism, we modify overflow system of the chemostat into the right system of Fig. 2. It can be regulated through replenishing water to reduce the concentration of the microorganisms when the concentration reaches a certain threshold which can be measured by optoelectronic devices and other ways.

We propose a mathematical model concerning the chemostat with variable yield and impulsive state feedback control according to the principle of the right system of Fig. 2.

In this paper, we will prove that the model without impulsive effect has a globally asymptotically stable positive equilibrium under certain condition. We will also discuss the existence and stability of periodic solution of the chemostat model with variable yield with impulsive state feedback control according to the existence criteria [12] and the stability theorem [26] of periodic solution of the general impulsive autonomous system. This paper is organized as follows. The model and some preliminary results are presented in the next section. The simple qualitative analysis of the system without impulsive effect is given in Section 3. In Section 4, the existence and stability of periodic solution of order one of differential equation with impulsive state feedback control are investigated. Numerical simulations are given in Section 5. Finally, Some conclusion and biological discussions are provided in Section 6.

2. The model and Preliminaries

The general model of continuously culturing microorganism in a chemostat is described by the following differential equation [27]:

$$\begin{cases} \dot{S} = D(S_{in} - S) - \frac{1}{\delta} f(S)x, \\ \dot{x} = f(S)x - Dx. \end{cases} \tag{2.1}$$

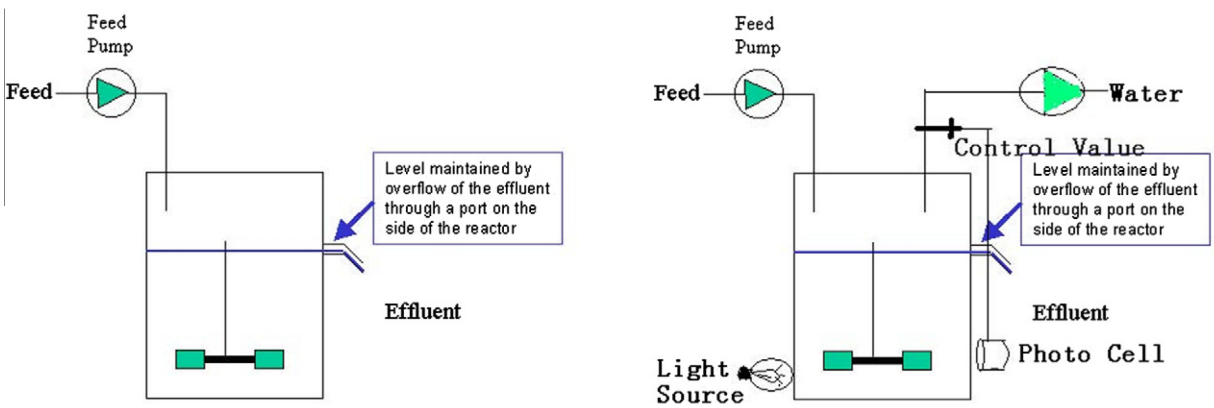

Fig. 2. The sketch map of chemostat and modified chemostat.