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Recursive computational formulas of the least squares criterion functions for scalar system identification ☆

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ABSTRACT

The paper discusses recursive computation problems of the criterion functions of several least squares type parameter estimation methods for linear regression models, including the well-known recursive least squares (RLS) algorithm, the weighted RLS algorithm, the forgetting factor RLS algorithm and the finite-data-window RLS algorithm without or with a forgetting factor. The recursive computation formulas of the criterion functions are derived by using the recursive parameter estimation equations. The proposed recursive computation formulas can be extended to the estimation algorithms of the pseudo-linear regression models for equation error systems and output error systems. Finally, the simulation example is provided.

1. Introduction

The parameter estimation is of great importance to system modeling and identification [1–5], adaptive control [6–10], signal processing [11,12]. Typical parameter estimation methods contain the iterative algorithms [13–17] and the recursive algorithms [18–20]. The iterative algorithms can be used to solve some matrix equations [21–28], such as the famous Jacobi iteration and the Gauss–Seidel iteration [29,30]. The recursive estimation algorithm can be used to on-line identify the parameters of systems and real-time update the parameters estimates at each step [31–33]. In the field of linear algebra, Xie et al. studied gradient based and least squares based iterative algorithms for linear matrix equations [34]; Dehghan and Hajarian presented two iterative algorithms for solving a pair of matrix equations $AYB = E$ and $CYD = F$ and the generalized coupled Sylvester matrix equations [35,36]; Ding et al. derived the iterative solutions to matrix equations of the form $A_iXB_i = F_i$ [37].

In the field of system identification, Wang et al. proposed a filtering based recursive least squares (RLS) identification algorithm for CARARMA systems [38] and an auxiliary model-based recursive generalized least squares parameter estimation algorithm for Hammerstein OEAR systems [39]; Ding and Chen studied the performance bounds of the forgetting factor least squares algorithm for time-varying systems with finite measurement data [40]; Ding and Xiao developed the

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finite-data-window recursive least squares algorithm with a forgetting factor for dynamical modeling (the FDW-FF-RLS algorithm for short) [41].

In general, a parameter estimation algorithm can be obtained by minimizing a quadratic cost function which is the square sum of the differences between the system outputs and the model outputs [42,43]. For online identification, the parameter estimation algorithm is implemented in a recursive form. Therefore, a natural question is how to compute the cost function in a recursive form since the values of the cost functions can measure the parameter estimation accuracy [44]. This is the work of this paper.

Recently, Ma et al. studied the recursive computational formulas of the criterion functions for the well-known weighted recursive least squares algorithm and the finite-data-window recursive least squares algorithm for linear regressive models [45] and the recursive computational relations of the cost functions for the least-squares-type algorithms for multivariable (or multivariate) linear regressive models [46]. On the basis of the work in [45,46], this paper derives the recursive computational formulas of the quadratic criterion functions for recursive least squares type parameter estimation algorithms, including the RLS algorithm in Section 2, the weighted RLS algorithm in Section 3, the forgetting factor RLS algorithm in Section 4, the finite-data-window RLS algorithm in Section 5 and the FDW-FF-RLS algorithm in Section 6. Section 7 simply discusses the recursive computational formulas of the criterion functions for multivariable equation error systems. Section 8 provides a numerical example to illustrate the proposed methods. Finally, concluding remarks are given in Section 9.

2. The recursive least squares algorithm

Let us introduce some notations first. "$A =: X$" or "$X := A$" stands for "A is defined as X"; the symbol $\mathbf{I}$ ($\mathbf{I}_n$) stands for an identity matrix of appropriate size ($n \times n$); the superscript T denotes the matrix transpose; the norm of a matrix $\mathbf{X}$ is defined by $\|\mathbf{X}\|^2 := \mathbf{tr}[\mathbf{X}\mathbf{X}^T]$; $\hat{\theta}(t)$ denotes the estimate of $\boldsymbol{\theta}$ at time t.

Recently, Ma and Ding studied the recursive relations of the criterion functions for the least squares parameter estimation algorithms for multivariable systems, including the multivariate RLS (MRLS) algorithm for multivariate linear regressive models, the forgetting factor MRLS algorithm and the finite-data-window MRLS algorithm with a forgetting factor (i.e., the FDW-FF-MRLS algorithm for short) and the FDW-FF-RLS algorithm for the multivariable controlled autoregressive models [46]. On the basis of the work in [46], this paper discusses simply the recursive computational formulas of the least squares criterion functions for scalar systems described by the following linear regressive models,

$$y(t) = \boldsymbol{\varphi}^T(t)\boldsymbol{\theta} + v(t), \tag{1}$$

where $y(t)$ is the system output, $v(t)$ is a white noise with zero mean, $\boldsymbol{\theta} \in \mathbb{R}^n$ is the parameter vector to be identified and $\boldsymbol{\varphi}(t) \in R^n$ is the regressive information vector consisting of the system inputs and outputs.

Consider the data set $\{y(i), \boldsymbol{\varphi}(i) : 1 \leqslant i \leqslant t\}$ and define a quadratic criterion function,

$$J_1(\boldsymbol{\theta}) := \sum_{j=1}^{t}[y(j) - \boldsymbol{\varphi}^T(j)\boldsymbol{\theta}]^2.$$

Define the innovation $e(t) := y(t) - \boldsymbol{\varphi}^T(t)\hat{\boldsymbol{\theta}}(t-1) \in \mathbb{R}^1$ [47–49]. Letting the derivative of $J_1(\boldsymbol{\theta})$ with respect to $\boldsymbol{\theta}$ be zero:

$$\left.\frac{\partial J_1(\boldsymbol{\theta})}{\partial \boldsymbol{\theta}}\right|_{\boldsymbol{\theta}=\hat{\boldsymbol{\theta}}(t)} = -2\sum_{j=1}^{t}\boldsymbol{\varphi}(j)[y(j) - \boldsymbol{\varphi}^T(j)\hat{\boldsymbol{\theta}}(t)] = \mathbf{0}, \tag{2}$$

we can obtain the following recursive least squares (RLS) algorithm for estimating $\boldsymbol{\theta}$ [1,47,50]:

$$\hat{\boldsymbol{\theta}}(t) = \hat{\boldsymbol{\theta}}(t-1) + \mathbf{L}(t)e(t), \tag{3}$$

$$e(t) = y(t) - \boldsymbol{\varphi}^T(t)\hat{\boldsymbol{\theta}}(t-1), \tag{4}$$

$$\mathbf{L}(t) = \mathbf{P}(t)\boldsymbol{\varphi}(t) = \frac{\mathbf{P}(t-1)\boldsymbol{\varphi}(t)}{1 + \boldsymbol{\varphi}^T(t)\mathbf{P}(t-1)\boldsymbol{\varphi}(t)}, \tag{5}$$

$$\mathbf{P}(t) = [\mathbf{I} - \mathbf{L}(t)\boldsymbol{\varphi}^T(t)]\mathbf{P}(t-1),\ \mathbf{P}(0) = p_0\mathbf{I}, \tag{6}$$

where $\mathbf{P}(t) \in R^{n\times n}$ denotes the covariance matrix, and p_0 is a large positive number.

The criterion function $J_1(\boldsymbol{\theta})$ with $\boldsymbol{\theta} = \hat{\boldsymbol{\theta}}(t)$ is given by

$$J_1[\hat{\boldsymbol{\theta}}(t)] = \sum_{j=1}^{t}[y(j) - \boldsymbol{\varphi}^T(j)\hat{\boldsymbol{\theta}}(t)]^2. \tag{7}$$