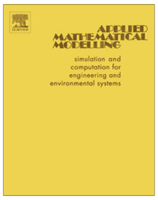
Applied Mathematical Modelling 38 (2014) 28–37

Contents lists available at SciVerse ScienceDirect

Applied Mathematical Modelling

journal homepage: www.elsevier.com/locate/apm



A general mathematical model for continuous generating machining of screw rotors with worm-shaped tools

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A R T I C L E I N F O

Article history:
Received 2 March 2012
Received in revised form 14 April 2013
Accepted 31 May 2013
Available online 24 June 2013

Keywords:
Screw rotor
Rotor hobbing
Worm-shaped tool
Generating machining

A B S T R A C T

Screw rotors play a crucial role in the performance of compressors. For the large-batch production of small- or medium-sized rotors, continuously rotor hobbing or grinding may be more efficient than form machining. In this study, a general mathematical model was developed for the generating machining of screw rotors with a worm-shaped tool. A two-parameter enveloping theory was applied to simulate the cutting process as the tool conducts polynomial feed motion considering its cutting edge. The normal errors of the generated cutting lines were computed and presented on the rotor tooth surface topologies to show the correctness and practicability of the proposed model.

1. Introduction

The features of the twin-screw compressor, one of the high-rotation positive-displacement compressors, include high reliability, ease of operation and maintenance, balanced transmission power, good adaptability, and the ability to transport various mixed liquids; thereby, the twin-screw compressor has been widely used in the refrigeration, air-conditioning, and chemical industries. The operation of a twin-screw compressor depends on the constant meshing rotation between the pair of conjugated rotors in the chamber, resulting in a progressive compression process attributed to the periodic change in spline displacement. Therefore, the rotor is the most crucial component of the twin-screw compressor, whereby the geometry and the machining precision of the rotors can greatly affect compressor performance.

The screw rotor is actually a cylindrical helical gear with a slightly complex gear profile. Although the machining of helical gears can be applied to the machining of screw rotors, each axial section profile of a rotor has equal clearance, whereas each axial section profile of a modified gear is different. The disk-type forming tool is mostly used in screw rotor machining, in which each spline is roughly rotary milled, grinding allowanced, and rotary finished by a disk-type forming wheel to meet the required surface clearance. However, this approach to rotor machining requires a longer manufacture time than the continuous, non-rotary generating machining (hobbing and worm grinding), and thereby is not economical for the large-batch production of small- or medium-sized rotors. Although hobbing machining have been used in batch of production of screw compressor rotors for more than thirty years, there are many issues being discussed, including how to solve cutting lines on rotor surfaces formed by hob cutting edges, reducing machining errors, saving on manufacture costs, and promoting geometric modifiability of the machined rotor. Therefore, the theory of generating machining and the motion of rotor hobbing could be further investigated.

Litvin [1] described various gears and tool designs in his book, “Theory of Gearing”, published in 1956. Based on gear theory, Litvin and Feng [2] proposed the geometric design and simulation of the meshing of rotors in a twin-screw compressor

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http://dx.doi.org/10.1016/j.apm.2013.05.056

in 1997. In 2003, Litvin et al. [3] presented modifications to the tooth surfaces of involute helical gears using form grinding and generating grinding. The two-parameter enveloping method was used to resolve issues in the machining surface in generating grinding. Surface contact and stress analysis simulations were also performed in that study. A more thorough description of the gear forming and hobbing tools can be found in the book by Litvin and Fuentes [4], published in 2004. These methods are extensively applied in this study to simulate the generating machining process of rotors.

In 1998, Yoshida et al. [5] proposed a method to obtain the thread surface of the screw rotor, which is cut using a hob with a basic thread profile given as a series of dispersive points. However, under certain circumstances, the singular point and the interference phenomenon may occur on the thread surface of the screw rotor. Xing [6] described the profile and geometry of twin-screw compressor rotors and evaluated the operating performance and design of the rotor tools. Xing suggested gear hobbing machining is superior to rotary milling machining, in terms of cutting speed and machining precision. Kaneko et al. [7] proposed a built-up hob for a screw-compressor rotor, which is composed of a hob head and cutter blades, with both the side and the end relief angle of the cutting edge. The cutting edge of the rack face can be re-ground and sharpened without changing the profile of the rack face. The rotors were fabricated by the built-up hob and measured by CMM. The pitch and profile error were about ±10 μm and ±20 μm. The improved, specialized rotor hob can save on tool manufacture costs, and the module-based tool can be re-sharpened, thus maintaining machining precision. Hashitani et al. [8] presented the Mitsubishi ZE15A gear grinding machine, which is used to finish quiet and small gears using the thread-shaped grinding worm. A new method of automated correction of the pressure angle has been developed without manually adjusting the setting angle of the dressing unit.

Stosic et al. [9] proposed a new N-Type rotor based on the predefined gear rack and gear enveloping method, and used it in the optimal design of various machines and testing. They also described the designs of the rotor hob and the forming tool, and later, Stosic [10] used the gear enveloping method to calculate the relative kinematic relationship of each point between the forming tool and the rotors under machining operation. Additionally, tool wear was estimated and verified via experimentation. Further, Wu and Fong [11] defined a normal rack comprising a specific curve based on the characteristics of twin-screw compressor rotors. The developed tool can be used in the machining of different helix angles of the rotor surface while maintaining the pressure angle of the rotor profile of the normal section constant. The tool can also be used to solve the generation of singular points, which is caused by the generating machining of the rotor surface.

As the references indicate, the techniques of roughing and finishing rotors with a worm-shaped tool were developed with an emphasis on the design of the rotor hob or the wheel profile of the worm-shaped tool, and less on the machining process simulation of rotors by the worm-shaped tool, as well as the numerical solution of cutting points and the different surface errors of rotors caused by the different machining motions are issues which need to be resolved. Further, generation of the rotor tooth profile with the generating machining of the worm-shaped tool was mostly simplified as a planar rack generation, which cannot reflect the practical machined tooth approach because the tool path is a function of time. Consequently, in this study, a general mathematical model is established for the generating machining of screw rotors with the worm-shaped tool. In addition, the relative moving coordinate system between the rotor and worm tool is built. The influence of tool feeds in the axial, radial and tangential directions on the generated rotor surface is investigated, and the generated rotor surface error is presented to check the correctness of the developed model. These may be helpful in predicting the tooth surface shape of the machined rotor while the moving path or speed of the worm-shaped tool is changed.

2. General mathematical model of continuous machining for screw rotors

2.1. Setting of center distance and shaft angle

As shown in Fig. 1, a general coordinate system for continuous machining of the screw rotor is presented, wherein the coordinate system S_w is affixed to the worm-shaped tool, rotating around the z_w axis with a rotation angle ϕ_w, coordinate

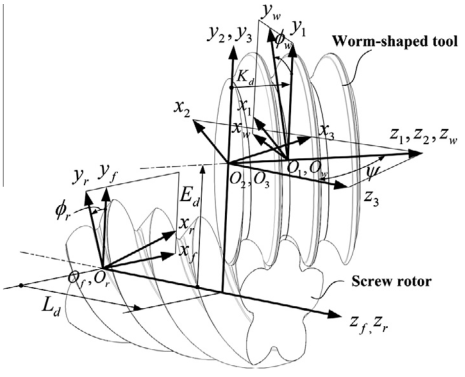

Fig. 1. General coordinate systems of the worm-shaped tool machining for screw rotors.