Applied Mathematical Modelling 38 (2014) 38–49

Contents lists available at SciVerse ScienceDirect

Applied Mathematical Modelling

journal homepage: www.elsevier.com/locate/apm

An iterative approach to solve multiobjective linear fractional programming problems

E. Valipour [a,*], M.A. Yaghoobi [a], M. Mashinchi [b]

[a] Department of Mathematics, Faculty of Mathematics and Computer, Shahid Bahonar University of Kerman, Kerman, Iran
[b] Department of Statistics, Faculty of Mathematics and Computer, Shahid Bahonar University of Kerman, Kerman, Iran

ARTICLE INFO

Article history:
Received 29 June 2012
Received in revised form 17 April 2013
Accepted 31 May 2013
Available online 24 June 2013

Keywords:
Multiple objective programming
Fractional programming
Efficiency
Iterative methods
Linear programming.

ABSTRACT

This paper suggests an iterative parametric approach for solving multiobjective linear fractional programming (MOLFP) problems which only uses linear programming to obtain efficient solutions and always converges to an efficient solution. A numerical example shows that this approach performs better than some existing algorithms. Randomly generated MOLFP problems are also solved to demonstrate the performance of new introduced algorithm.

1. Introduction

A multiobjective fractional programming (MOFP) problem optimizes several ratio objective functions over a feasible region. It is called a multiobjective linear fractional programming (MOLFP) problem when the numerators and denominators of the objective functions are linear and the feasible region is a polyhedron. Fractional objectives appear in many real world situations. For instance, we often need to optimize the efficiency of some activities like cost/time, cost/profit, and output/employee. For an overview of these applications, we refer to [1,2] and the references therein.

Fractional objectives with convex numerators and denominators are not convex and even linear fractional objectives are only quasiconvex [2]. Thus, solving MOFP problems or even MOLFP problems is usually difficult and sometimes finding even one efficient solution is of importance. Kornbluth and Steuer [3] presented a simplex based algorithm to find a weakly efficient set of MOLFP problems. They also used a goal programming approach for MOLFP problems [4]. Nykowski and Zolkiewski [5] proposed a compromise procedure to obtain extreme efficient solutions. Metev and Gueorguieva [6] used nonlinear programming for finding a weakly efficient set of solutions. Caballero and Hernandez [7] computed a discrete estimation of weakly efficient set of solutions. Cambini et al. [8] reviewed methods for solving biobjective linear fractional programming. In addition, many researchers investigated solving sum of linear ratios problems by iterative algorithms [9–14].

Dinkelbach [15] used a parametric approach to solve linear fractional programming problems. Several authors extended Dinkelbach's approach to solve several problems involving fractional objectives such as generalized fractional programming problems [16,17] and the minimum spanning tree with sum of ratios problems [18]. Almogy and Levin [19] extended the parametric approach of Dinkelbach [15] to solve sum of ratios problems. However, Falk and Palocsay [20] showed that

* Corresponding author. Tel.: +98 341 3202269; fax: +98 341 3201080.
E-mail addresses: valipour@math.uk.ac.ir, ezat.valipour@gmail.com (E. Valipour), yaghoobi@uk.ac.ir (M.A. Yaghoobi), mashinchi@uk.ac.ir (M. Mashinchi).

0307-904X/$ - see front matter
http://dx.doi.org/10.1016/j.apm.2013.05.046

the approach in [19] does not always lead to appropriate solutions. In [12], they extended the parametric approach of Dinkelbach to solve sum of ratios, product of ratios and product of linear functions. Tammer et al. [21] also extended Dinkelbach's approach to solve MOLFP problems. In their approach, estimating the parameters was based on solving some equations. However, their approach does not necessarily guarantee an efficient solution. Using the parametric approach of Dikelbach, Gomes et al. [22] introduced an MOLP problem with a weighted version to deal with an MOLFP problem. However, they only give some optimality conditions. Although Dinkelbach's approach has been used to solve many different problems involving fractional objectives, there is no absolutely successful extension to solve MOLFP problems.

The current paper attempts to propose an iterative algorithm that extends Dinkelbach's approach [15] to solve MOLFP problems. It considers a parametric linear objective function and inserts new constraints so that the convergency of the algorithm is guaranteed. Also, if each objective is a ratio of a convex (concave) function over a concave (convex) one, and the feasible region is convex, then in each iteration a convex (concave) problem must be solved and convergency still holds. Since the proposed algorithm is iterative, it is compared with the iterative technique presented by Costa [9]. The technique of Costa [9] is interesting since it performs well in comparison with the existing techniques such as those of Kuno [13], Phuong and Tuy [14] and Dai et al. [11]. It must be mentioned that Costa and Alves [10] presented another iterative method that is slightly different from Costa [9]. This paper presents a brief survey of Costa's technique [9].

The rest of the paper is organized as follows: Section 2 introduces some notation, definitions, and basic theorems. The techniques of Costa [9] and Costa and Alves [10] are stated briefly in Section 3. Section 4 presents the main results of our algorithm and the proof of its convergency. In Section 5, the proposed technique is compared with Costa's technique and some examples are presented. Finally, Section 6 is devoted to some concluding remarks.

2. Preliminaries

The MOLFP problem can be formulated as [3]:

$$\begin{array}{ll} \max & Z(x) = \left(z_1(x), \ldots, z_p(x)\right)^t \\ s.t. & x \in X = \{x \in \mathbb{R}^n : Ax \leqslant b,\ x \geqslant 0\}, \end{array} \tag{1}$$

where

X is compact, $A \in \mathbb{R}^{m\times n}, b \in \mathbb{R}^m$, $z_i(x) = \frac{N_i(x)}{D_i(x)} = \frac{a_i x + b_i}{c_i x + d_i}$ ($a_i, c_i \in \mathbb{R}^n, b_i$, $d_i \in \mathbb{R}$ for $i = 1, \ldots, p$), and t stands for transpose.

It is customary to assume that $D_i(x) = c_i x + d_i > 0, \forall x \in X$. Moreover, since $X \subset \mathbb{R}^n$, then $\mathbb{R}^n$ is called the decision space. Let $Y = Z(X) = \{Z(x) = (z_1(x), \ldots, z_p(x))^t : x \in X\}$, then since $Y \subset \mathbb{R}^p$, $\mathbb{R}^p$ is called the objective space.

In Problem (1), the optimal solution for one function is not necessarily optimal for the other functions, and hence the notions of "efficient solutions", "ε-efficient solutions", and "weakly efficient solutions" are introduced [23].

Definition 2.1 (*See for instance [23]*). A point $\bar{x} \in X$ is an efficient solution of Problem (1) if there is no other $x \in X$ such that $z_i(x) \geqslant z_i(\bar{x})$ for all $i = 1, \ldots, p$ and $z_j(x) > z_j(\bar{x})$ for at least one j.

Definition 2.2 (*See for instance [24]*). Let $0 < \epsilon_i \in \mathbb{R}$ for $i = 1, \ldots, p$ and set $\varepsilon = (\epsilon_1, \ldots, \epsilon_p)$. A point $\bar{x} \in X$ is an ε-efficient solution of Problem (1) if there is no other $x \in X$ such that $z_i(x) \geqslant z_i(\bar{x}) + \epsilon_i$ for all $i = 1, \ldots, p$ and $z_j(x) > z_j(\bar{x}) + \epsilon_j$ for at least one j.

Definition 2.3 (*See for instance [23]*). A point $\bar{x} \in X$ is a weakly efficient solution of Problem (1) if there is no other $x \in X$ such that $z_i(x) > z_i(\bar{x})$ for all $i = 1, \ldots, p$.

It must be noticed that the notions of "efficient solutions", "ε-efficient solutions", and "weakly efficient solutions" are usually used in the decision space, and their images in the objective space are called "nondominated solutions", "ε-nondominated solutions", and "weakly non-dominated solutions", respectively. Although some authors use the expression "nondominated solution" instead of "efficient solution" [9,10], in the current research we distinguish between them. Therefore, we use the notions of "efficient" and "nondominated" for the decision space and the objective space, respectively. Also, we denote the set of all nondominated solutions of Problem (1) by $Y_N = \{Z(x) \in Y : x \text{is an efficient solution of Problem}(1)\}$.

Definition 2.4 (*See for instance [23]*). The set Y_N is called externally stable if for each $Z(x) \in Y \setminus Y_N$ there exists $Z(\bar{x}) \in Y_N$ such that $z_i(\bar{x}) \geqslant z_i(x)$ for all $i = 1, \ldots, p$ and $z_j(\bar{x}) > z_j(x)$ for at least one j.

Lemma 2.1. *If Y is a compact set, then Y_N is externally stable.*

Proof. The proof is a straightforward result of Theorem 2.21 of [23]. □

Moreover, a straightforward result of Lemma 2.1 is as follows: