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Sensitivity analysis in interval-valued trapezoidal fuzzy number linear programming problems

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ABSTRACT

The aim of this article is to introduce a formulation of fuzzy linear programming problems involving the level (h^L, h^U)-interval-valued trapezoidal fuzzy numbers as parameters. Indeed, such a formulation is the general form of trapezoidal fuzzy number linear programming problems. Then, it is demonstrated that study of the sensitivity analysis for the level (h^L, h^U)-interval-valued trapezoidal fuzzy number linear programming problems gives rise to the same expected results as those obtained for trapezoidal fuzzy number linear programming problems.

1. Introduction

Fuzzy set theory was first applied to conventional linear programming problems by Zimmerman [1]. Following this attempt, fuzzy linear programming has been developed and extended in a number of directions with successful applications. Among the various methods proposed for solving fuzzy number linear programming problems, the method based on the concept of comparison of fuzzy numbers by the help of ranking functions is one of the most convenient [2–6].

Nowadays, sensitivity analysis is one of the interesting researches in fuzzy linear programming. The first attempt to study sensitivity analysis for fuzzy linear programming problems is due to Hamacher et al. [7] and later followed by others [8–10].

In fuzzy set theory, all of the fuzzy objective functions and the fuzzy constraints are represented by their corresponding membership functions. There are various types of membership functions in existing fuzzy sets fields that can be divided into several patterns, such as linear [11], piecewise linear [12], hyperbolic [13], exponential and hyperbolic inverse [14], *S*-curve [15], Gaussian curve [16], generalized bell-shaped [17], *Z*-curve [18], etc. One of the most frequently applied form of membership functions is the linear form which is used here to model a level (h^L, h^U)-interval-valued trapezoidal fuzzy number linear programming problem.

The majority of studies for handling fuzzy linear programming focus on the problems whose parameters are in form of trapezoidal fuzzy numbers. Researchers have rarely considered the problems involving generalized interval-valued trapezoidal fuzzy numbers.

In this article, we introduce a formulation of fuzzy linear programming problems where the level (h^L, h^U)-interval-valued trapezoidal fuzzy numbers are regarded as parameters. Indeed, such a formulation is the general form of fuzzy number linear programming considered by Ebrahimnejad [2] and others [3–6,19]. Next, we propose a method for solving the level (h^L, h^U)-interval-valued trapezoidal fuzzy number linear programming problems based on the comparison of the level

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(h^L, h^U)-interval-valued trapezoidal fuzzy numbers by the help of signed distance ranking. Finally, we show that study of the sensitivity analysis for the level (h^L, h^U)-interval-valued trapezoidal fuzzy number linear programming problems gives rise to the expected results as same as those obtained in [2].

This paper is organized as follows. In Section 2, we review some definitions and axioms which are used in the analysis throughout this contribution. Section 3 is devoted to introducing a level (h^L, h^U)-interval-valued trapezoidal fuzzy number linear programming problem and to present the primal simplex algorithm in fuzzy sense for solving such a problem. The main results concerning sensitivity analysis for the level (h^L, h^U)-interval-valued trapezoidal fuzzy number linear programming problems are given in Section 4. In Section 5, the major advantages and disadvantages of the proposed method over the existing methods are discussed. This paper is concluded in Section 6.

2. Preliminaries and fundamental concepts

This section is devoted to review some necessary background and notions of the level (h^L, h^U)-interval-valued trapezoidal fuzzy numbers which are used throughout this study.

Definition 2.1 [20]. A level h-trapezoidal fuzzy number $\tilde{A}$, denoted by $\tilde{A} = (a_1, a_2, a_3, a_4; h), 0 < h \leqslant 1$, is a fuzzy set on R with the membership function as

$$\tilde{A}(x) = \begin{cases} h\frac{(x-a_1)}{(a_2-a_1)}, & a_1 \leqslant x \leqslant a_2, \\ h, & a_2 \leqslant x \leqslant a_3, \\ h\frac{(a_4-x)}{(a_4-a_3)}, & a_3 \leqslant x \leqslant a_4, \\ 0, & otherwise. \end{cases} \tag{1}$$

Let $F_{TN}(h)$ be the family of all level h-trapezoidal fuzzy numbers, that is,

$$F_{TN}(h) = \{\tilde{A} = (a_1, a_2, a_3, a_4; h) : a_1 \leqslant a_2 \leqslant a_3 \leqslant a_4\}, \quad 0 < h \leqslant 1.$$

Definition 2.2 [21]. Let $\tilde{A}^L \in F_{TN}(h^L)$ and $\tilde{A}^U \in F_{TN}(h^U)$. A level (h^L, h^U)-interval-valued trapezoidal fuzzy number $\tilde{\tilde{A}}$, denoted by

$$\tilde{\tilde{A}} = [\tilde{A}^L, \tilde{A}^U] = \langle (a_1^L, a_2^L, a_3^L, a_4^L; h^L), (a_1^U, a_2^U, a_3^U, a_4^U; h^U) \rangle \tag{2}$$

is an interval-valued fuzzy set on R with the lower trapezoidal fuzzy number $\tilde{A}^L$ expressing by

$$\tilde{A}^L(x) = \begin{cases} h^L \frac{(x-a_1^L)}{(a_2^L-a_1^L)}, & a_1^L \leqslant x \leqslant a_2^L, \\ h^L, & a_2^L \leqslant x \leqslant a_3^L, \\ h^L \frac{(a_4^L-x)}{(a_4^L-a_3^L)}, & a_3^L \leqslant x \leqslant a_4^L, \\ 0, & otherwise \end{cases} \tag{3}$$

and the upper trapezoidal fuzzy number $\tilde{A}^U$ expressing by

$$\tilde{A}^U(x) = \begin{cases} h^U \frac{(x-a_1^U)}{(a_2^U-a_1^U)}, & a_1^U \leqslant x \leqslant a_2^U, \\ h^U, & a_2^U \leqslant x \leqslant a_3^U, \\ h^U \frac{(a_4^U-x)}{(a_4^U-a_3^U)}, & a_3^U \leqslant x \leqslant a_4^U, \\ 0, & otherwise, \end{cases} \tag{4}$$

where $a_1^L \leqslant a_2^L \leqslant a_3^L \leqslant a_4^L, a_1^U \leqslant a_2^U \leqslant a_3^U \leqslant a_4^U, 0 < h^L \leqslant h^U \leqslant 1$, $a_1^U \leqslant a_1^L$ and $a_4^L \leqslant a_4^U$. Moreover, $\tilde{A}^L \subseteq \tilde{A}^U$.

Let $F_{IVTN}(h^L, h^U)$ be the family of all level (h^L, h^U)-interval-valued trapezoidal fuzzy numbers, that is,

$$\begin{aligned} F_{IVTN}(h^L, h^U) = &\Big\{\tilde{\tilde{A}} = [\tilde{A}^L, \tilde{A}^U] = \langle (a_1^L, a_2^L, a_3^L, a_4^L; h^L), (a_1^U, a_2^U, a_3^U, a_4^U; h^U) \rangle : \tilde{A}^L \in F_{TN}(h^L), \tilde{A}^U \in F_{TN}(h^U), a_1^U \leqslant a_1^L, a_4^L \leqslant a_4^U\Big\}, \\ & 0 < h^L \leqslant h^U \leqslant 1. \end{aligned} \tag{5}$$

Remark 2.1. If $h^L = h^U = 1$, then $\tilde{\tilde{A}}$ is a normal interval-valued trapezoidal fuzzy number. If $\tilde{A}^L = \tilde{A}^U$, then $\tilde{\tilde{A}}$ becomes a generalized trapezoidal fuzzy number. If $a_2^L = a_3^L$ and $a_2^U = a_3^U$, then $\tilde{\tilde{A}}$ is a level (h^L, h^U)-interval-valued triangular fuzzy number. If $a_2^L = a_3^L, a_2^U = a_3^U$ and $h^L = h^U = 1$, then $\tilde{\tilde{A}}$ is a normal interval-valued triangular fuzzy number. If $a_1^L = a_2^L = a_3^L = a_4^L, a_1^U = a_2^U = a_3^U = a_4^U$ and $h^L = h^U = 1$, then $\tilde{A}^L$ and $\tilde{A}^U$ become crisp values and therefore $\tilde{\tilde{A}}$ is a crisp interval.