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Kink solutions for three new fifth order nonlinear equations

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ABSTRACT

In this work we examine three new fifth order nonlinear evolution equations. The simplified form of the Hirota's direct method is used to derive multiple kink solutions for the first two (1+1)-dimensional equations, and only two soliton solutions for the third (2+1)-dimensional equation. The dispersion relation is the same for the first two equations whereas the third one possesses a different dispersion relation.

1. Introduction

In the context of nonlinear evolution equations, studies are flourishing because these equations are able to describe the real features in a variety of science and engineering areas. Moreover, studies of finding soliton solutions of the nonlinear equations attracted huge number of works in a variety of fields. Towards this goal, a variety of powerful methods to construct multiple soliton solutions has been established in the fields of mathematical physics and engineering. Examples of the methods that have been used are the Hirota bilinear method [1], the simplified Hirota's method [2–6], the Bäcklund transformation method, Darboux transformation, Pfaffian technique, the inverse scattering method, the Painlevé analysis, the generalized symmetry method, the subsidiary ordinary differential equation method, the coupled amplitude-phase formulation, the sine–cosine method, the sech–tanh method, the mapping and the deformation approach, and many other methods [7–10]. The Hirota's bilinear method [1], and the simplified Hirota's method developed in [2,10] are rather heuristic and significant to handle equations with constant coefficients. These two methods possess powerful features that make it practical for the determination of single soliton and multiple soliton solutions for a wide class of nonlinear evolution equations. The Hereman-Nuseir method [2] does not depend on the construction of the bilinear forms, instead it assumes the multi–soliton solutions can be expressed as polynomials of exponential functions [2–6]. The power of the Hereman–Nuseir method has been emphasized by employing it to several nonlinear evolution equation in [10]. The computer symbolic systems such as Maple and Mathematica allow us to perform complicated and tedious calculations.

In science and engineering, scientific phenomena give a variety of solutions that are characterized by distinct features. Traveling waves appear in many distinct physical structures in solitary wave theory such as solitons, kinks, peakons, cuspons, and compactons and many others [5–10]. Solitons are localized traveling waves which are asymptotically zero at large distances. In other words, solitons are localized wave packets with exponential wings or tails. Solitons are generated from a robust balance between nonlinearity and dispersion. Solitons exhibit properties typically associated with particles. Kink waves are solitons that rise or descend from one asymptotic state to another, and hence another type of traveling waves as in the case of the Burgers hierarchy. Peakons, that are peaked solitary wave solutions, are another type of travelling waves

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as in the case of Camassa-Holm equation. For peakons, the traveling wave solutions are smooth except for a peak at a corner of its crest [10]. Peakons are the points at which spatial derivative changes sign so that peakons have a finite jump in first derivative of the solution. Cuspons are other forms of solitons where solution exhibits cusps at their crests. Unlike peakons where the derivatives at the peak differ only by a sign, the derivatives at the jump of a cuspon diverges. The compactons [10], which are solitons with compact spatial support such that each compacton is a soliton confined to a finite core or a soliton without exponential tails or wings. Other types of travelling waves arise in science such as negatons, positons and complexitons.

It is well known that a new classification of exact and explicit solutions of soliton equations, both without and with self–consistent sources, is usually used. This classification depends mainly on the property of the associated spectral parameters λ of the Lax pair of each equation. Negaton solution is related to the negative spectral parameter, i.e for $\lambda < 0$. Negaton [8,9] solutions, like solitons, contain exponential functions of the space variable, but not trigonometric functions. Positon solution is related to the positive spectral parameter, i.e for $\lambda > 0$. Positon solution [8,9] is slowly decreasing and oscillating. Unlike soliton and negaton, positon solution contains trigonometric functions of the space variable but not exponential functions. However, the complexiton solution is related to the complex spectral parameter [6]. Complexiton solution [8,9] is usually expressed by the combinations of trigonometric functions and hyperbolic functions of the space variable. We recall that if the spectral parameter is zero, i.e for $\lambda = 0$, the resulting solution is a soliton expressed in terms of exponential functions.

Usually, the interactions between soliton solutions obtained by Backlund transformation method, Darboux transformation, Painlevé integrability, etc. for integrable models are considered to be completely elastic [1–9]. By elastic interactions we mean that if a soliton meets another soliton of its kind, they interact, but without destroying each other's identities. That is to say, the amplitude, velocity and wave shape of a soliton do not change after the non-linear interaction [7–9]. However, as for some soliton models, completely non-elastic interactions will occur when specific conditions between the wave vectors and velocities are satisfied such as in compactons. For instance, at a specific time, one soliton may fission to two or more solitons; or on the contrarily, two or more solitons will fusion to one soliton [6–8]. We call these two types of phenomena soliton fission and soliton fusion respectively. In fact, for many real physical models (such as in organic membrane and macromolecule material [9], in SrBaNi oxidation crystal and waveguide [9], in even-clump DNA and in many physical fields like plasma physics, nuclear physics, hydrodynamics and so on, people have observed the phenomena soliton fusion and soliton fission.

In this work, our main focus will be on extending our previous works in [3,4], where the following new fifth order nonlinear evolution equations of the form

$$u_{ttt} - u_{txxxx} - 4(u_x u_t)_{xx} - 4(u_x u_{xt})_x = 0 \tag{1}$$

and a new generalized form of (1)

$$u_{ttt} - u_{txxxx} - \alpha(u_x u_t)_{xx} - \beta(u_x u_{xt})_x = 0, \tag{2}$$

where α and β were investigated. Multiple soliton solutions for (1) and (2) were derived in [3,4].

To extend the aforementioned equations, we introduce three new fifth-order extensions in the form

$$u_{ttt} - u_{txxxx} - u_{txx} - 4(u_x u_t)_{xx} - 4(u_x u_{xt})_x = 0, \tag{3}$$

$$u_{ttt} - u_{txxxx} - u_{txx} - 4(u_x u_{xt})_x = 0 \tag{4}$$

and the (2+1)-dimensional equation

$$u_{ttt} - u_{tyyyy} - u_{txx} - 4(u_y u_{yt})_y = 0. \tag{5}$$

Eq. (3) is derived by adding $-u_{txx}$ to (1). Eq. (4) is obtained from deleting the fourth term of from (3). However, the (2+1)-dimensional Eq. (5) is obtained from (3) where the spatio-temporal derivative u_{txxxx} is changed to u_{tyyyy} and using the term $-4(u_y u_{yt})_y$ instead 0f $-4(u_x u_{xt})_x$.

It is interesting to note that in [5] we also extended the fifth-order Eqs. (1) and (2) to higher-dimensional fifth-order equations. The new (2+1)-dimensional and (3+1) dimensional fifth-order equations read

$$u_{ttt} - u_{txxxx} - u_{tyyyy} - \alpha(u_x u_{xt})_x = 0 \tag{6}$$

and

$$u_{ttt} - u_{txxxx} - u_{tyyyy} - u_{tzzzz} - \beta(u_x u_{xt})_x = 0, \tag{7}$$

respectively. In [5], it was shown that the extended forms (6) and (7) presented multiple soliton solutions of distinct physical structures, in particular distinct dispersion relation and distinct phase shifts, compared to the solutions obtained in [3,4].

Unlike our works in [3–5], the work in [6] handles a modified KdV type equation that reads

$$uu_{xxt} - u_x u_{xt} - 4u^3 u_t + 4uu_{xxx} - 4u_x u_{xx} - 16u^3 u_x = 0. \tag{8}$$

This equation is of third-order and not derived or related to any of the aforementioned fifth-order equations. This equation was found to be peculiar and admits a variety of travelling wave solutions: kinks, solitons, peakons, etc.