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Discontinuous cellular automaton method for crack growth analysis without remeshing

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A B S T R A C T

A numerical technical of discontinuous cellular automaton method for crack growth analysis without remeshing is developed. In this method, the level set method is employed to track the crack location and its growth path, where the level set functions and calculation grids are independent, so no explicit meshing for crack surface and no remeshing for crack growth are needed. Then, the discontinuous enrichment shape functions which are enriched by the Heaviside function and the exact near-tip asymptotic field functions are constructed to model the discontinuity of cracks. Finally, a discontinuous cellular automaton theory is proposed, which are composed of cell, neighborhood and updating rules for discontinuous case. There is an advantage that the calculation is only applied on local cell, so no assembled stiffness matrix but only cell stiffness is needed, which can overcome the stiffness matrix assembling difficulty caused by unequal degrees of nodal freedom for different cells, and much easier to consider the local properties of cells. Besides, the present method requires much less computer memory than that of XFEM because of it local property.

Combined level set method, the discontinuous enrichment shape functions and discontinuous cellular automaton theory, the discontinuous cellular automaton method is proposed, which can conveniently achieve the analysis from continuity to discontinuity. Numerical examples are given to illustrate that the present method is effective, and can be further extended into practical engineering.

1. Introduction

Failures in engineering structure are usually caused by some defects, such as cracks, inclusion and interface and so on, and many failures result largely from the microcracks which grow beyond a safety limit. While the cracks in engineering structures are always exist during manufacturing and service, and exact solutions for most of complex cracks growth are not available, so crack growth simulation is a challenging and important problem in practical engineering.

Crack is a discontinuous structure, and its propagation leads to the expansion of some local areas of structure from continuous into discontinuous, which brings some difficulty for many numerical methods. Finite element method has been firstly used to calculate stress intensity factors [1,2]. But it requires the element edges to coincide with the crack surface, and remeshing is inevitable when the discontinuous surface changes. Later, boundary element method [3,4], boundary integral equation method [5] and boundary collocation method [6,7], in which the mesh is only enforced on the boundary, have

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been proposed to solve the fracture calculation, and those methods avoid a large part of remeshing because only the boundary is needed to be meshed.

Recently, the element-free Galerkin method [8–10] has been applied to fracture computation, and the essential feature of this methods is that they only require a set of nodes to construct the calculation model. Besides, meshless local Petrov–Galerkin method [11] and local boundary integral equation method [12] have been also applied to analyze growing crack problems. Additionally, numerical manifold method [13] has been also used to simulate the crack propagation, which is suitable to simulate continuous and discontinuous problems at the same frame, but its dual grid property causes some difficulty in some complex crack growth.

Recently, some methods in FE framework without remeshing have been developed. Using partition of unity [14], Belytschko and Black [15] first introduced a method for solving crack problem with FE framework. Later, Moes et al. [16] used the Heaviside function to describe the discontinuity across crack faces and developed crack tip enrichments. After that, a junction function concept has been introduced to solve multiple branched cracks [17], and extended finite element method (XFEM) has been developed in detail. Based on XFEM formulation, 2D and 3D crack growth analysis with or without contact friction have been developed by Sukumar et al. [18] and Dolbow et al. [19]. Besides, Xiao and Karihaloo [20] discussed the influence of quadrature rules on the accuracy of XFEM.

Incorporating the application of the analytical or numerical function into tradition FE approximation with the partition of unity, generalized finite element method [21–23] has been proposed to solve fracture mechanics, which can improve the local and global accuracy of numerical solutions. Those methods have been widely used to simulate crack growth problems, because the finite element mesh can be completely independent of the morphology of the model, and the crack surface and crack front are completely independent of the mesh, so no remeshing is needed in crack propagation simulation.

In order to track complex crack configurations, the level set method(LSM) was developed by Osher and Sethian [24], which was used for tracking the moving interface. Then the LSM was used to describe the topology changes of the interface. Later, Belytschko et al. [25] and Stolarska et al. [26] combined the LSM with the XFEM to study the growth of a fatigue crack and several frictionless contact problems. With the use of the LSM, the grid for XFEM is completely independent of the crack faces, so no remeshing is need for crack growth analysis.

It is known that the node freedoms are different for different nodes for XFEM, which brings some difficulty for global stiffness matrix assembling, besides, it takes a large amount of computer memory for this procedure. So the cellular automaton (CA) theory is used to overcome this defect. The cellular automaton (CA) theory was initially derived from the self-organization theory in biology. Shen et al. [27] developed elastic updating rules and applied it to solve the solid mechanical problem. Gurdal and Tatting [28] built a lattice model to solve the plane lattice deformation problem, additionally Hopman and Leamy [29], Leamy [30] developed an application of cellular automata modeling to elastodynamics problem and arbitrary two-dimensional geometries, and further Feng et al. [31] used the lattice CA model to simulate the failure process of heterogeneous rocks. There are two advantages of the application of CA model, one is the calculation only applied on local cell, so no assembled stiffness matrix but only cell stiffness is needed, which can overcome the stiffness matrix assembling difficulty caused by unequal degrees of nodal freedom for different cells, and much easier to consider the local property of node and element. Another is that it can be easily extended to the large-scale simulation for its easy implementation of the parallel algorithm. Besides, much less computer memory requirement is achieved because of it local property.

As the discontinuous numerical methods, DEM [32] and DDA [33] are widely used in soil and rock engineering. Those two methods are based block theory, which can simulate the moving, rotating, opening and clogging of rock blocks. When those methods are applied to simulate the crack growth, its growth path can only be along to the block boundary, and its fracture cannot extend into the blocks, but we do not know the direction of the propagation of crack before its growth. So those two methods are suitable for solving the known discontinuous structure problems, and can not accurately achieve the expansion from continuity to discontinuity.

In this work, discontinuous enrichment shape function, level set method and discontinuous cellular automaton theory are combined, and a numerical technical of discontinuous cellular automaton method is proposed, in which the calculation is only applied on local elements and nodes, and no assembled stiffness matrix is needed, so it is much easier to consider the local property of material and its interaction. Firstly, the level set method is employed to track the crack location and its growth path, in which the level set functions and calculation grid are independent, so no explicit meshing for crack surface and no remeshing for crack growth are needed. Then, the discontinuous enrichment shape functions which are enriched by the Heaviside function and the exact near-tip asymptotic field functions are constructed to model the discontinuity of cracks. Finally, a discontinuous cellular automaton theory is proposed, in which the calculation is only applied on local cell, so no assembled stiffness matrix but only cell stiffness is needed.

2. Continuous and discontinuous structure modeling

Crack is a strong discontinuous structure, and in the traditional finite element the crack surface should coincide with the element edge in order to model the strong discontinuous displacement and stress field. In this work, approximation of the discontinuous displacement field is based on a specially designed shape functions, in which the Heaviside function is used to simulate the discontinuity and the exact near-tip asymptotic field functions is employed to model the high gradient stress field near the crack tip.