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Non-linear buckling for the surface rectangular delamination of laminated piezoelectric shells

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ABSTRACT

An analytical method is presented to study the non-linear buckling characteristic of rectangular local delamination near the surface of fiber-reinforced piezoelectric lamination shells under coupled mechanical and electric loads. The stacking sequence of fiber reinforced lamination shells with piezoelectric layers is considered as symmetry, but the stacking sequence of rectangular local delamination sub-shells is arbitrary. Based on the nonlinear displacement mode of delaminated sub-shells, the effects of electric fields, the geometrical, physical parameters and stacking sequences of piezoelectric laminated shells on the non-linear local buckling for rectangular delamination near the surface of piezoelectric laminated base-shells are solved.

1. Introduction

Because the inherent electromechanical effect of piezoelectric materials has many important industrial applications, the use of piezoelectric materials have been broadly carried out for the sensitive element and active control of light weight smart structures [1–6]. Due to the coupled electromechanical behaviors of piezoelectric materials and their availability in the form of thin sheets, composite laminated structures with piezoelectric layers are extensively used in aerospace, pressure vessel, and shipbuilding areas [7–13]. The sensitive and active control functions of piezoelectric laminated structures can be affected by defects in piezoelectric laminated structural, such as local delamination induced by manufacturing process, tool drop, bird strike and debris hit, so that the stiffness behaviors and the load-carrying capacity of the piezoelectric laminated structure are drastically reduced [14,15]. Utilizing the benefits of direct and converse piezoelectric effects, integrated piezoelectric sensor and actuator layers bonded in fiber reinforced laminated structures may be used to adequately control the deflection, shape and buckling of the laminated structure [16,17]. Kima et al. [12] presented an investigation into the buckling of an orthotropic piezoelectric rectangular laminate with weak interfaces based on the state-space formulations from the three-dimensional theory of elasticity.

The local delamination buckling of laminated structures can make the strength and stiffness of the laminated structures reduce [18,19]. Aiello and Ombres [20] investigated local buckling characteristics of sandwich panels made with laminated faces. Overall and local buckling of sandwich plates with laminated faceplates is investigated in Ref. [21]. Utilizing a spline finite strip method, Azhari et al. [22] investigated local buckling of composite laminated plate assemblies. A study on elliptical delamination buckling in a symmetrical laminate under complex load is presented in Ref. [23]. Authors [24] presented an asymptotic analytical method to solve delamination buckling and growth in layered plates. Shivakumar et al. [25] and Timoshenko et al. [26] presented an analytical solution for the buckling of a sub-laminate in a quasi-isotropic base laminate.

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The previous researches mainly reported the investigations on the global buckling of piezoelectric laminated structures or the buckling of local delamination in fiber-reinforced lamination structures subjected to only mechanical loading.

A study on elliptically delaminated buckling near the surface of piezoelectric laminated shells under electric and thermal loads is presented in Ref. [27]. The calculation model and solution process for locally buckling of delamination in laminated structures are dependent on the shape of delamination. The investigation on the locally buckling for rectangular delaminations near the surface of piezoelectric laminated shells in this paper is yet necessary to understood the effect of rectangular delaminations on the buckling performance, desired life and strength of piezoelectric laminated shells because the shapes of delamination near the surface of laminated structures may be various. The study on the nonlinear local buckling for rectangular delamination near the surface of piezoelectric laminated shells subjected to coupling electric and mechanical loads has been not reported in literatures, so far.

This paper presents an analytical method to solve the nonlinear local buckling for rectangular delamination of piezoelectric laminated cylindrical shells subjected to electric field and axial loading. The transverse displacement and deflection in the middle plane of rectangular delamination are considered as non-linear displacement modes, the stacking sequence of piezoelectric laminated base-shells is considered as symmetry, the stacking sequence of locally rectangular delamination is arbitrary, and the geometrical axes of rectangular delamination may be inconsistent with the geometrical main axes of base-shells. It is well known that internal/external pressure or axial, torsional, or bending loads are, generally, considered to investigate the global buckling performance of intact laminated shells. However, the local buckling for delaminated sub-shells in laminated base-shells is easily induced by axial loading (strain or force). In order to simplify the solution process of local buckling for delaminated sub-shells in laminated piezoelectric base-shells, the constant axial strain is considered in the present model.

The critical axial strains of nonlinear buckling for local rectangular delamination near the surface of piezoelectric laminated shells subjected to electric field and axial loading are obtained by utilizing the energy method [26], and some discussions are given.

2. Calculation model and solution method

The calculation model of piezoelectric laminated shells (base-shells) with local rectangular delamination is shown in Fig. 1, where $h_b/R \ll 1$ and $h_s/h_b \ll 1$, R and h_b are, respectively, the mid-surface radius and the thickness of piezoelectric laminated base-shells, and h_s is the thickness of local rectangular delamination sub-shells.

The geometrical coordinate (Global system) of piezoelectric lamination shells and the geometrical coordinate (Local system) of rectangular delamination are described by (X,Y,Z) and (x,y,z) respectively, where φ represents the angle between the global geometrical axes X and local geometrical x, and θ represents the angle between the material's main axis (L) of the laminated piezoelectric structures and the axis X.

U, V, W and U', V', W' represent, respectively, the mid-surface displacements of laminated piezoelectric shells in the global geometrical axes X, Y, Z and in the local geometrical axes x, y, z. The mid-surface strains of laminated piezoelectric shells in the global geometrical axes (X,Y,Z) are expressed as

$$\varepsilon_X = dU/dX, \quad \gamma_{XY} = dV/dX,$$

$$\varepsilon_y = [2\pi(R-W) - 2\pi R]/(2\pi R) = -W/R. \tag{1}$$

From shallow shell theory [26], when the displacement field of local rectangular delamination near the surface of laminated piezoelectric shells in the local geometrical axes x, y, z is defined as u, v, w, respectively, and the mid-surface radius, R', of local rectangular delamination satisfies $R' \approx R \gg w$, the mid-surface strains, curvatures and twist curvature of the rectangular delamination can be written as

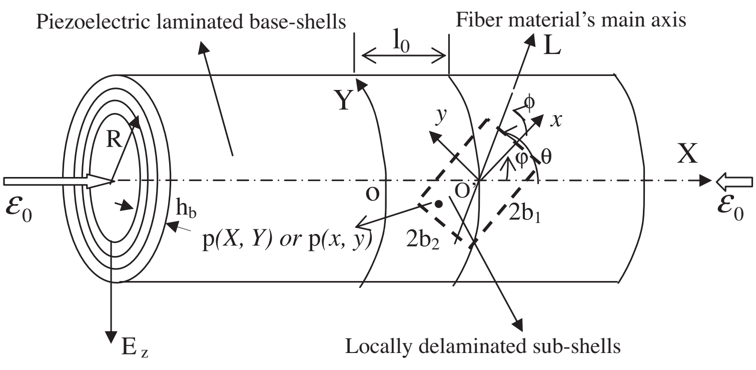

Fig. 1. Calculation model and geometrical axes of laminated piezoelectric base-shells and local rectangular delaminations, where E_z is the radial electric field strength exerted on piezoelectric layers.