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Short communication

Single-machine ready times scheduling with group technology and proportional linear deterioration

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ABSTRACT

Scheduling research has increasingly taken the concept of deterioration into consideration. In this paper, we study a single machine group scheduling problem with deterioration effect, where the jobs are already put into groups, before any optimization. We assume that the actual processing times of jobs are increasing functions of their starting times, i.e., the job processing times are described by a function which is proportional to a linear function of time. The setup times of groups are assumed to be fixed and known. For some special cases of minimizing the makespan with ready times of the jobs, we show that the problem can be solved in polynomial time for the proposed model. For the general case, a heuristic algorithm is proposed, and the computational experiments show that the performance of the heuristic is fairly accurately in obtaining near-optimal solutions. The results imply that the average percentage error of the proposed heuristic algorithm from optimal solutions is less than 3%.

1. Introduction

In recent years, a growing interest in studying scheduling problems with time-dependent processing times (deteriorating jobs), have been award of, i.e., jobs whose processing times are increasing functions of their starting times. Job deterioration appears, e.g., in scheduling maintenance jobs, steel production, cleaning assignments or emergency medicine, where the jobs that are processed later will take longer times to process. An extensive survey of different scheduling problems with time-dependent processing times (deteriorating jobs) can be found in Gawiejnowicz [1]. For details on time-dependent scheduling problems without group technology assumption, the reader may refer to the papers by Wang and Guo [2], Wang and Wang [3], Zhao and Tang [4], Moslehi and Jafari [5], Wang et al. [6], Sun et al. [7], Wang et al. [8], Wei et al. [9], Wang and Wang [10], Wang et al. [11], Wang and Wang [12], Rachaniotis and Pappis [13], Voutsinas and Pappis [14], Huang and Wang [15], Ng et al. [16], Sun et al. [17], Wang et al. [18], and Wang and Wang [19].

In the second place, in group technology, it is conventional to schedule continuously all jobs from the same group. Group technology that groups similar products into families helps increase the efficiency of operations and decrease the requirement of facilities (Mitrofanov [20], Potts and Van Wassenhove [21], Kovalyov and Janiak [22], Webster and Baker [23]). To the best of our knowledge, only few results concerning scheduling models and problems with time-dependent processing times (deteriorating jobs) and group technology simultaneously are known. Wang et al. [24] considered single machine

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group scheduling where the setup times of groups are constant, and the actual processing time of a job is a general linear decreasing function of its starting time. They showed that the makespan minimization problem and total completion time minimization problem can be solved in polynomial time. Wu et al. [25], Wu and Lee [26], Wang et al. [27], and Wang et al. [28] considered a situation where the group setup times and job processing times are both described by a linear deterioration function. For the simple linear deterioration, Wu et al. [25] proved that the makespan minimization problem and the total completion minimization problem can be solved in polynomial time. For the linear deterioration function with identical deterioration rate, Wu and Lee [26] proved that the makespan minimization problem remains polynomially solvable. They also showed that the sum of completion times problem is polynomially solvable when the numbers of jobs in each group are equal. For the proportional linear deterioration, Wang et al. [27] proved that the makespan minimization problem and the total weighted completion time minimization problem can be solved in polynomial time. For a general deterioration function, Wang et al. [28] proved that the makespan minimization problem can be solved in polynomial time. Huang et al. [29] and Bai et al. [30] considered single machine scheduling problems with learning effects and deteriorating jobs. Wang et al. [31] considered a single machine group scheduling problem, in which the processing time of a job is a simple linear function of its starting time, and the setup time of a group is assumed to be known and fixed. For a special case, they proved that the makespan minimization problem with ready times of the jobs is solvable in polynomial time.

In this paper, we continue the work of Wang et al. [31], by considering a more general deterioration model that includes the one given in [31] as a special case. The rest of the paper is organized as follows: In Section 2 we formulate the model. In Sections 3 we consider some special cases of the makespan minimization problem and solve the problem in polynomial time. In Section 4, we propose a heuristic algorithm for the general case and give the computational experiments. In the final section, we conclude the paper.

2. The model

We will use the following notation:

$m (m \geqslant 2)$	the number of groups
n	the total number of jobs
G_i	group $i, i = 1, 2, \ldots, m$
n_i	the number of jobs belonging to group G_i
J_{ij}	job j in group $G_i, i = 1, 2, \ldots, m, j = 1, 2, \ldots, n_i$
r_{ij}	the ready (arrival) time of job $J_{ij}, i = 1, 2, \ldots, m, j = 1, 2, \ldots, n_i$
s_i	the setup time of group G_i
p_{ij}	the actual processing time of job J_{ij}
α_{ij}	the deterioration rate of job J_{ij}
π	a job sequence of n jobs
$C_{ij} = C_{ij}(\pi)$	the completion time of job J_{ij} in π
$C_{\max} = \max\{C_{ij} \mid i = 1, 2, \ldots, m, j = 1, 2, \ldots, n_i\}$	the makespan of a given permutation

There are n jobs grouped into m groups, and our problem is to schedule these n jobs on a single machine in order to minimize the performance measure of makespan (i.e., the maximum completion time of all jobs). We assume that the processing of a job may not be interrupted. Let n_i be the number of jobs belonging to group G_i ($n_1 + n_2 + \ldots + n_m = n$). In addition, J_{ij} denotes the jth job in group G_i, $r_{ij} \geqslant 0$ denotes the ready (arrival) time of job J_{ij}, p_{ij} denotes the actual processing time of job J_{ij}, s_i denotes the setup time of group G_i required if the machine switches from one group to another and all setup times of groups for processing at time $t_0 \geqslant 0$. Note that the groups' set up times are independent of the previous and the next group, if any. As in Kononov and Gawiejnowicz [32], we consider the following proportional linear deterioration model

$$p_{ij} = \alpha_{ij}(a + bt),$$

where $a \geqslant 0, b \geqslant 0, \alpha_{ij}$ is the deterioration rate of job J_{ij}, and t is its start time.

Let GT indicate that the problem is a scheduling problem with group technology. Adopting the three-field notation for scheduling problem introduced by Graham et al. [33], we denote the above problem as $1|r_{ij}, p_{ij} = \alpha_{ij}(a + bt), s_i, GT|C_{\max}$.

3. Makespan minimization scheduling problem for some special cases

In this section, we consider a single machine makespan minimization scheduling problem under the proposed model. For some special cases, we will show that the problem can be solved in polynomial time.