Applied Mathematical Modelling 38 (2014) 392–402

Contents lists available at SciVerse ScienceDirect

Applied Mathematical Modelling

journal homepage: www.elsevier.com/locate/apm

Short communication

The influences of longitudinal surface roughness on sub-critical and super-critical limit cycles of short journal bearings

Jaw-Ren Lin *

Department of Mechanical Engineering, Taoyuan Innovation Institute of Technology, No. 414, Sec. 3, Jhongshan E. Rd., Jhongli 320, Taiwan

ARTICLE INFO

Article history:
Received 15 October 2012
Received in revised form 10 May 2013
Accepted 24 June 2013
Available online 4 July 2013

Keywords:
Journal bearings
Longitudinal roughness
Sub-critical bifurcation
Super-critical bifurcation

ABSTRACT

By applying the stochastic model of rough surfaces by Christensen (1969–1970, 1971) [1,2] together with the Hopf bifurcation theory by Hassard et al. (1981) [3], the present study is mainly concerned with the influences of longitudinal roughness patterns on the linear stability regions, Hopf bifurcation regions, sub-critical and super-critical limit cycles of short journal bearings. It is found that the longitudinal rough-surface bearings can exhibit Hopf bifurcation behaviors in the vicinity of bifurcation points. For fixed bearing parameter, the effects of longitudinal roughness structures provide an increase in the linear stability region, as well as a reduction in the size of sub-critical and super-critical limit cycles as compared to the smooth-bearing case.

1. Introduction

Rotating journal bearings are the basis of mechanical devices in engineering sciences. Fundamentally, the basic study on the steady-state performance of hydrodynamic journal bearings has been observed by Cameron [4], Hamrock [5] and Lin and Hwang [6]. The film pressure and the load capacity are predicted. To realize the stiffness and damping performances, the linear dynamic characteristics of journal bearings have been discussed by Holmes [7], Lund [8] and Lin and Hwang [9]. On the other hand, the hydrodynamic journal bearings may develop an oil-whirl instability under certain operating conditions. When the whirl amplitude grows too large, it may endanger and damage the bearing system. Therefore, the nonlinear stability analysis has been investigated by applying different approaches, such as the numerical transient method by Badgley and Booker [10] and Akers et al. [11]; and the Hopf bifurcation theory by Lin and Hwang [12] and Wang and Khonsari [13]. However, all these contributions concentrate on the bearing characteristics with the ideal assumption that the bearing surfaces are perfectly smooth. In engineering practice, bearing surfaces are generally rough owing to the manufacturing process that has been used in various forming and finishing stages. Therefore, the effects of surface roughness become important when the size of the surface asperity height reaches the same order as the lubricant film thickness. Therefore, Christensen [1,2] developed a stochastic approach to predict the effects of surface roughness. The averaged Reynolds-type equation is derived and applied to investigate the performance characteristics of rough bearings. Based on this stochastic theory, a lot of studies of surface roughness effects on bearing performances have been carried out, for example, the hydrostatic thrust bearings by Lin [14]; the squeeze film bearings by Lin et al. [15]; the steady-state journal bearings by Gururajan and Prakash [16]; the linear dynamic journal bearings by Chiang et al. [17]. In addition, the effects of isotropic roughness patterns by Lin [18] and of transverse roughness structures by Lin [19] on the limit cycles of journal bearings have also been investigated by applying the Hopf bifurcation theory of Hassard et al. [3]. According to their results, the roughness structures of isotropic and

* Tel.: +886 3 4361070; fax: +886 3 4384670.
E-mail address: jrlin@tiit.edu.tw

0307-904X/$ - see front matter
http://dx.doi.org/10.1016/j.apm.2013.06.024

transverse patterns have significant influences on the weakly nonlinear stability regions and the size of limit cycles of journal bearings. Since the study of longitudinal roughness effects on the weakly nonlinear stability characteristics is absent, a further investigation is motivated.

By applying the stochastic model of rough surfaces by Christensen [1,2] together with the Hopf bifurcation theory by Hassard et al. [3], the influences of longitudinal roughness patterns on the Hopf bifurcation characteristics are investigated in the present paper. Bearing performances (including the linear stability regions, Hopf bifurcation regions, sub-critical and super-critical limit cycles) are presented and discussed in comparison with rough bearings of the isotropic structures by Lin [18] and the transverse structures by Lin [19].

2. Formulation

The physical cross-section geometry of a longitudinal rough-surface short journal bearing at the mid-plane $\bar{z} = 0$ is described in Fig. 1. The length of the bearing is $\bar{L}$, and the inner journal with radius $\bar{r}_j$ rotates within the outer bearing housing with an angular velocity $\bar{\Omega}$. Assumed that the incompressible Newtonian fluid flow in the film region is laminar and isothermal, and the thin-film lubrication theory is applicable in the present study. According to the derivation of Cameron [4] and Hamrock [5], the dynamic modified Reynolds equation for the local film pressure $\bar{p}_L$ of a short bearing is expressed as:

$$\frac{\partial}{\partial \bar{z}}\left[\overline{H}_L^3 \frac{\partial \bar{p}_L}{\partial \bar{z}}\right] = 6\mu\left(\overline{\Omega} - 2\frac{d\beta}{d\bar{t}}\right)\frac{\partial \overline{H}_L}{\partial \theta} + 12\mu \frac{\partial \overline{H}_L}{\partial \bar{t}}, \tag{1}$$

where μ is the lubricant viscosity, β is the attitude angle, θ is the circumferential coordinate, and $\bar{t}$ denotes the time. For the bearing with longitudinal roughness patterns, the local film thickness $\overline{H}_L$ is made up of two parts:

$$\overline{H}_L = \bar{h}(\theta, \bar{t}) + \bar{\delta}(\bar{z}, \bar{\xi}), \tag{2}$$

where $\bar{h}$ represents the nominal smooth part of the film geometry, $\bar{\delta}$ describes the random part resulting from the surface roughness evaluated from the nominal smooth height, and $\bar{\xi}$ denotes a random variable describing the surface pattern of longitudinal roughness. Applying the Christensen stochastic theory [1,2], the stochastic dynamic modified Reynolds equation for the bearing taking into account the influences of longitudinal rough surfaces can be written as:

$$\frac{\partial}{\partial \bar{z}}\left[\frac{1}{E(\overline{H}_L^{-3})}\frac{\partial E(\bar{p}_L)}{\partial \bar{z}}\right] = 6\mu\left(\bar{\Omega} - 2\frac{d\beta}{d\bar{t}}\right)\frac{\partial [E(\overline{H}_L)]}{\partial \theta} + 12\mu \frac{\partial [E(\overline{H}_L)]}{\partial \bar{t}}. \tag{3}$$

In this equation, $\bar{p} = E(\bar{p}_L)$ denotes the mean hydrodynamic film pressure, $E(a)$ defines the expectancy operator,

$$E(a) = \int_{\bar{\delta}=-\infty}^{+\infty} a \cdot f(\bar{\delta}) d\bar{\delta}, \tag{4}$$

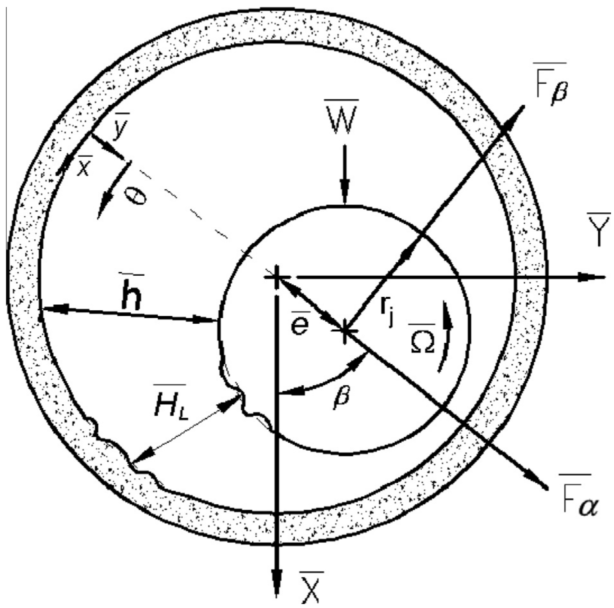

Fig. 1. Physical cross-section geometry of a longitudinal rough-surface short journal bearing at the mid-plane $\bar{z} = 0$.