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Short communication

Combined state and least squares parameter estimation algorithms for dynamic systems ☆

Feng Ding *

Key Laboratory of Advanced Process Control for Light Industry (Ministry of Education), Jiangnan University, Wuxi 214122, PR China
Control Science and Engineering Research Center, Jiangnan University, Wuxi 214122, PR China

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ABSTRACT

The control theory and automation technology cast the glory of our era. Highly integrated computer chip and automation products are changing our lives. Mathematical models and parameter estimation are basic for automatic control. This paper discusses the parameter estimation algorithm of establishing the mathematical models for dynamic systems and presents an estimated states based recursive least squares algorithm, and the states of the system are computed through the Kalman filter using the estimated parameters. A numerical example is provided to confirm the effectiveness of the proposed algorithm.

1. Introduction

Numerical methods have wide applications for solving matrix equations or compute the model parameters of dynamic systems [1–3]. Typical numerical identification methods include the gradient search, the least squares and the Newton methods [4–6]. Parameter estimation is basic for controller design [7–9], filtering and state estimation [10,11] and system identification [12–14]. Recently, a gradient based iterative method and a least squares based iterative method were presented for identifying multiple-input multiple-output systems [15] and for identifying Wiener nonlinear systems [16]; and a Newton recursive and a Newton iterative algorithms were developed for identifying Hammerstein nonlinear systems [17]; a least squares based recursive estimation algorithm and a least squares based iterative algorithm were proposed for output error moving average systems using data filtering [18]; several maximum likelihood based recursive least squares algorithms were discussed for systems with colored noises [19–21].

In the area of parameter estimation [22–25], Zhang et al. proposed a bias compensation based recursive least squares method for stochastic systems with colored noises [26] and for a class of multiple-input single-output systems [27]; Liu et al. discussed multi-innovation stochastic gradient approach for multiple-input single-output systems using the multi-innovation identification theory and the auxiliary model identification idea [28] and analyzed the convergence of the stochastic gradient algorithm for multivariable ARX-like systems [29]. Ding et al. presented an auxiliary model based multi-innovation stochastic gradient algorithm for systems with scarce measurements [30] and an auxiliary model based recursive least squares algorithm for missing-data systems [31]. Xiao et al. presented a residual based interactive least squares

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* Address: Control Science and Engineering Research Center, Jiangnan University, Wuxi 214122, PR China.
E-mail address: fding@jiangnan.edu.cn

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algorithm for controlled autoregressive moving average systems [32]; Ding and Duan proposed a two-stage parameter estimation algorithms for Box–Jenkins systems [33].

In the field of state space system identification, Ding et al. presented a hierarchical identification method for the lifted state space model of general dual-rate systems [34] and for non-uniformly sampled-data systems [35]; Gu et al. discussed a least squares numerical parameter estimation algorithm for a state space model with multi-state delays, assuming the states of the system are available [36], and studied parameter and state estimation for a state space model with a one-unit state delay [37] and for a multivariable state space system with d-step state-delay [38]. This paper studied the identification method of canonical state space systems, assuming the states of the system are unavailable.

This paper is organized as follows. Section 2 derives the identification model for state space systems. Section 3 gives the parameter and state estimation algorithm. Section 4 provides an example to verify the effectiveness of the proposed algorithm. Finally, concluding remarks are given in Section 5.

2. The identification model for the state space systems

Let us define some notations. "$A =: X$" or "$X := A$" stands for "A is defined as X". Let z denote a unit forward shift operator with $z\boldsymbol{x}(t) = \boldsymbol{x}(t+1)$ and $z^{-1}\boldsymbol{x}(t) = \boldsymbol{x}(t-1)$.

Consider the following observer canonical state space system,

$$\boldsymbol{x}(t+1) = \boldsymbol{A}\boldsymbol{x}(t) + \boldsymbol{b}u(t), \tag{1}$$

$$y(t) = \boldsymbol{c}\boldsymbol{x}(t) + v(t), \tag{2}$$

where $\boldsymbol{x}(t) := [x_1(t), x_2(t), \cdots, x_n(t)]^{\mathrm{T}} \in \mathbb{R}^n$ is the state vector, $u(t) \in \mathbb{R}$ is the system input, $y(t) \in \mathbb{R}$ is the system output, $v(t) \in \mathbb{R}$ is random noise with zero mean, $\boldsymbol{A} \in \mathbb{R}^{n\times n}, \boldsymbol{b} \in \mathbb{R}^n$ and $\boldsymbol{c}$ are the system parameter matrix and vectors:

$$\boldsymbol{A} = \begin{bmatrix} -a_1 & 1 & 0 & \cdots & 0 \\ -a_2 & 0 & 1 & & \vdots \\ \vdots & \vdots & & \ddots & 0 \\ -a_{n-1} & 0 & \cdots & 0 & 1 \\ -a_n & 0 & \cdots & \cdots & 0 \end{bmatrix} \in \mathbb{R}^{n\times n}, \quad \boldsymbol{b} := \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_{n-1} \\ b_n \end{bmatrix} \in \mathbb{R}^n,$$

$$\boldsymbol{c} := [1, 0, 0, \ldots, 0] \in \mathbb{R}^{1\times n}.$$

The parameters $a_i \in \mathbb{R}$ and $b_i \in \mathbb{R}$ are to be identified from observation data $\{u(t), y(t) : t = 1, 2, 3, \cdots\}$.

From (1), we have

$$\begin{bmatrix} x_1(t+1) \\ x_2(t+1) \\ \vdots \\ x_{n-1}(t+1) \\ x_n(t+1) \end{bmatrix} = \begin{bmatrix} -a_1 & 1 & 0 & \cdots & 0 \\ -a_2 & 0 & 1 & & \vdots \\ \vdots & \vdots & & \ddots & 0 \\ -a_{n-1} & 0 & \cdots & 0 & 1 \\ -a_n & 0 & \cdots & \cdots & 0 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ \vdots \\ x_{n-1}(t) \\ x_n(t) \end{bmatrix} + \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_{n-1} \\ b_n \end{bmatrix} u(t), \tag{3}$$

$$y(t) = [1, 0, 0, \ldots, 0]\boldsymbol{x}(t) + v(t), \tag{4}$$

which can be written as

$$x_i(t+1) = -a_i x_1(t) + x_{i+1}(t) + b_i u(t), \quad i = 1, 2, \ldots, (n-1), \tag{5}$$

$$x_n(t+1) = -a_n x_1(t) + b_n u(t), \tag{6}$$

$$y(t) = x_1(t) + v(t). \tag{7}$$

Multiplying (5) by z^{-i} gives

$$x_i(t-i+1) = -a_i x_1(t-i) + x_{i+1}(t-i) + b_i u(t-i), \quad i = 1, 2, \ldots, (n-1).$$

Summing for i from $i = 1$ to $i = (n-1)$ gives

$$\sum_{i=1}^{n-1} x_i(t-i+1) = -\sum_{i=1}^{n-1} a_i x_1(t-i) + \sum_{i=1}^{n-1} x_{i+1}(t-i) + \sum_{i=1}^{n-1} b_i u(t-i),$$

or

$$x_1(t) = -\sum_{i=1}^{n-1} a_i x_1(t-i) + x_n(t-n+1) + \sum_{i=1}^{n-1} b_i u(t-i). \tag{8}$$