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Application of lattice Boltzmann method for incompressible viscous flows

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ABSTRACT

Because of the presence of corner eddies that change in number and pattern the lid-driven cavity problem has been found suitable to study various aspects of the performance of solution algorithms for incompressible viscous flows. It retains all the difficult flow physics and is characterized by a large primary eddy at the centre and secondary eddies located near the cavity corners. In this work, lid-driven cavity flow is simulated by lattice Boltzmann method with single-relaxation-time and it is compared with those by lattice Boltzmann method with multi-relaxation-time and finite difference method. The effects of the Reynolds number on the size, centre position and number of vortices are studied in detail together with the flow pattern in the cavity. The close agreement of the results bears testimony to the validity of this relatively new approach. However lattice Boltzmann method with multi-relaxation-time model is seen to remove the difficulties faces by the lattice Boltzmann method with single-relaxation-time at higher Reynolds numbers.

1. Introduction

The lid-driven cavity flow problem consisting in an incompressible viscous flow in a cavity whose top wall moves with a uniform velocity in its own plane has long been used for evaluating numerical techniques for the solution of incompressible viscous flows [1]. This lid-driven cavity problem is attractive because of its importance in industrial applications such as coating & drying technologies, melt spinning processes and many others [2]. It is also known that, the rectangular lid-driven cavity flow is an idealized representation of several engineering situations, such as the flow over cutouts, designs and repeated slots on the walls of heat exchangers or on the surface of aircraft bodies [3]. Because of its popularity, a plethora of experimental and numerical results are readily available for this problem in the literature [4–6].

The first major studies of the steady two-dimensional lid-driven cavity flow are due to Burggraf [7] for the square cavity and Pan and Acrivos [8] for other geometrical aspect ratios. In most of the available literature, the cavity flow field calculation is based on solving the Navier–Stokes equation using a finite difference method (FDM), finite volume method (FVM) and finite element method (FEM). In fact, as hundreds of papers attest, the lid-driven cavity flow problem is one of the benchmark problems used to test new computational schemes. In the last one and a half decade or so lattice Boltzmann method (LBM) has emerged as a new and effective approach of computational fluid dynamics (CFDs) and it has achieved considerable success in simulating fluid flows and heat transfer [9–11].

Many researchers carried out simulations of lid-driven square cavity flow by lattice Boltzmann method with Single-Relaxation-Time (LBM-SRT) model [12–15]. Hou et al. [12] extensively studied viscous flow in a lid-driven square cavity for a wide range of Reynolds number using LBM-SRT model. He found some ripples in the streamline, vorticity and pressure contour plots for high Reynolds number. Some of the other notable works in cavity flow by the LBM-SRT include those of Lai

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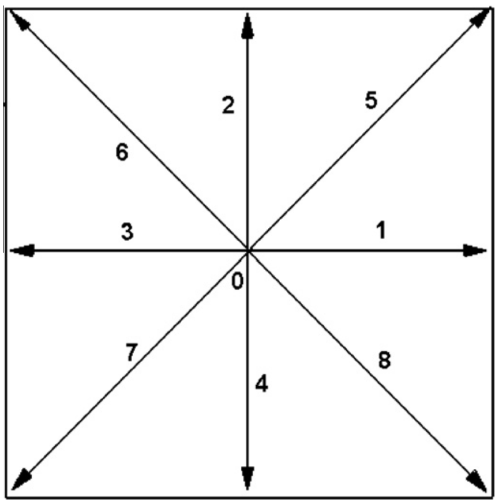

Fig. 1. Two-dimensional nine-velocity square lattice model.

et al. [13], Perumal and Dass [14], and Patil et al. [15]. From the literature, it is found that most of the work deals with lattice Boltzmann method with single-relaxation-time (LBM-SRT) and bounce-back boundary condition to study the cavity flow field. There appears to be very little work done on rectangular cavities (deep and shallow cavity) by continuum based methods and lattice boltzmann method with multi-relaxation-time (LBM–MRT) model, although they are of theoretical interest.

The main objective of the present work is to describe a detailed implementation of the LBM models and to demonstrate the validity of the LBM models at high Reynolds numbers. Therefore, the lid-driven cavity problem with different aspect ratios is studied in an effort to evaluate the performance of the LBM and to produce accurate steady state solutions for different values of the Reynolds number. This paper is organized in four sections. In Section 2 numerical methods including LBM-SRT, LBM–MRT and FDM is described. In Section 3 the lid-driven cavity problem with aspect ratio is described and the results are presented and validated. Concluding remarks are made in Section 4.

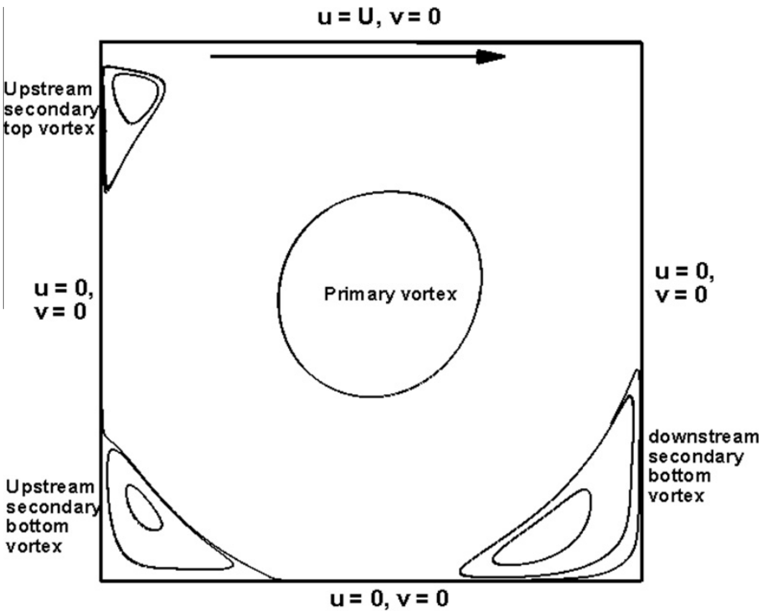

Fig. 2. Geometry of a single lid-driven cavity flow with boundary conditions.