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LQR based improved discrete PID controller design via optimum selection of weighting matrices using fractional order integral performance index

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ABSTRACT

The continuous and discrete time Linear Quadratic Regulator (LQR) theory has been used in this paper for the design of optimal analog and discrete PID controllers respectively. The PID controller gains are formulated as the optimal state-feedback gains, corresponding to the standard quadratic cost function involving the state variables and the controller effort. A real coded Genetic Algorithm (GA) has been used next to optimally find out the weighting matrices, associated with the respective optimal state-feedback regulator design while minimizing another time domain integral performance index, comprising of a weighted sum of Integral of Time multiplied Squared Error (ITSE) and the controller effort. The proposed methodology is extended for a new kind of fractional order (FO) integral performance indices. The impact of fractional order (as any arbitrary real order) cost function on the LQR tuned PID control loops is highlighted in the present work, along with the achievable cost of control. Guidelines for the choice of integral order of the performance index are given depending on the characteristics of the process, to be controlled.

1. Introduction

Classical optimal control theory has evolved over decades to formulate the well known Linear Quadratic Regulators which minimizes the excursion in state trajectories of a system while requiring minimum controller effort [1]. This typical behavior of LQR has motivated control designers to use it for the tuning of PID controllers [2,3]. PID controllers are most common in process industries due to its simplicity, ease of implementation and robustness. Using the Lyapunov's method, the optimal quadratic regulator design problem reduces to the Algebraic Riccati Equation (ARE) which is solved to calculate the state feedback gains for a chosen set of weighting matrices. These weighting matrices regulate the penalties on the deviation in the trajectories of the state variables (x) and control signal (u). Indeed, with an arbitrary choice of weighting matrices, the classical state-feedback optimal regulators seldom show good set-point tracking performance due to the absence of integral term unlike the PID controllers. Thus, combining the tuning philosophy of PID controllers with the concept of LQR allows the designer to enjoy both optimal set-point tracking and optimal cost of control within the same design framework.

Optimal control theory has been extended to tune PID controllers in few recent literatures. In Choi and Chung [4], an inverse optimal PID controller is designed considering the error and its integro-differential as the state variables, similar

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to the approach, presented in this paper. In Arruda et al. [5], a custom cost function has been minimized with GA to design multi-loop PID controllers as the weighted sum of ITSE and variance of the manipulated variable and controlled variable. PID controller tuning with state-space approach using the error and its first and second order derivative has been investigated in [6,7]. The method proposed LQR-PID of He et al. [2,3] has been extended for first and second order systems with zeros in the process model in Ghartemani et al. [8]. Ochi and Kondo [9] have shown that the integral type optimal servo for second order system can be reduced to a LQR problem and an optimal I-PD controller can be designed with this technique. Several classical optimal and robust control approaches of PID controller can be cast into a Linear Matrix Inequality (LMI) problem as in Ge et al. [10] which consider the controlled variable, its rate and integral of error as the state variables.

Genetic algorithm and other stochastic global optimization techniques have also been employed for various optimal control problems. Wang et al. [11] used GA to optimally find out the weighting matrices of LQR i.e., Q and R with a specified structure. The concept of GA based optimum selection of weighting matrices has been extended for LQR as well as pole placement problems in Poodeh et al. [12]. GA based optimal time domain [13] and frequency domain loop-shaping [14] based PID tuning problems are also popular in the contemporary research community. The mixed H_2/H_∞ optimal PID controller tuning of Chen et al. [14] has been improved with GA as a single objective disturbance rejection PID controller in Krohling and Rey [15] and as multi-objective loop-shaping based design in Lin et al. [16]. A wide class of standard optimal control problems has been solved using evolutionary and swarm intelligence based global optimization techniques in Ghosh et al. [17,18].

Fractional order systems and controllers are becoming increasingly popular in the automation and process control community. A state of the art survey on the design and application of fractional order system and controllers can be found in [19]. For optimum set-point tracking control of PID/FOPID controllers, time domain performance index optimization based tuning techniques are more popular and have been applied in Cao et al. [20], Das et al. [21] and Pan et al. [22,23]. The impact of choosing the weighting matrices of LQR are discussed by Saif [24] in a detailed manner. The present methodology selects the weighting matrices for the quadratic regulator design similar to that in [11,12], using Genetic Algorithm while minimizing a suitable time domain performance index. Then a new arbitrary (fractional) order integral performance index has been used as the objective function of GA, as suggested by Romero et al. [25] for signal processing applications. The impact of these new FO integral indices based PID design on the closed loop control performance as well as the corresponding optimality of the quadratic regulators are also highlighted in the present work. An analog PID controller and its discretized form a digital PID both have been tuned with the proposed optimum weight selection based corresponding continuous and discrete time LQR techniques for second order systems with very low and high damping as two illustrative examples.

The rest of the paper is organized as follows. Section 2 discusses about the theoretical framework for LQR based optimal analog and digital PID controller design. Section 3 proposes the GA based optimum weight selection methodology for LQR tuning of PID controllers. Section 4 validates the proposed argument with two classes of second order systems as two illustrative examples. The paper ends with the conclusion as Section 5, followed by the references.

2. Formulation of LQR based optimal PID controller for second order systems

2.1. Tuning of PID controllers as continuous time Linear Quadratic Regulators

He et al. [2,3] has given a formulation for tuning over-damped or critically-damped second order systems having two real open loop process poles. The concept has been extended in this sub-section for lightly damped processes as well. Also, in [2], it has been suggested that one of the real poles needs to be cancelled out by placing one of the controller zeros at the same position on the negative real axis of complex s-plane. Thus the second order plant to be controlled with a PID controller can be reduced to a first order process to be controlled by a PI controller. Indeed, this approach of He et al. [2] does not hold for lightly damped processes having oscillatory open loop dynamics as such reduction in not possible in this case. With the approach of optimal PID tuning for second order processes in [2], also the provision of simultaneously and optimally finding the three parameters of a PID controller (i.e., K_p, K_i, K_d) is lost that has been addressed in this paper. The present approach assumes the error, its rate and integral as the state variables and designs the optimal state-feedback controller gains as the PID controller parameters (Fig. 1).

In Fig. 1, a PID controller in parallel form (with proportional, integral and derivative gains as K_p, K_i, K_d) has been considered to control a second order system with known open loop damping ratio and natural frequency i.e., ξ^{ol}, ω_n^{ol} respectively. If the feedback control system is excited with an external input $r(t)$ to get a control signal $u(t)$ and output $y(t)$, then let us define the state variables as:

$$x_1 = \int e(t)dt, \quad x_2 = e(t), \quad x_3 = \frac{de(t)}{dt}. \tag{1}$$

From the block diagram presented in Fig. 1, it is clear that

$$\frac{Y(s)}{U(s)} = \frac{K}{s^2 + 2\xi^{ol}\omega_n^{ol}s + \left(\omega_n^{ol}\right)^2} = \frac{-E(s)}{U(s)}. \tag{2}$$