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A complementary tool to enhance the effectiveness of existing methods for heterogeneous fixed fleet vehicle routing problem

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ABSTRACT

The heterogeneous fixed fleet vehicle routing problem (HFFVRP) is a variant of the standard vehicle routing problem (VRP), in which the vertices have to be served using a fixed number of vehicles that could be different in size and fixed or variable costs. In this article, we propose an integer linear programming-based heuristic approach in order to solve the HFFVRP that could be used as a complementary tool to improve the performance of the existing methods of solving this problem. Computational results show the effectiveness of the proposed method.

1. Introduction

The standard vehicle routing problem (VRP) [1], is to satisfy the demand of a given set of vertices using a fleet of homogenous vehicles with side constraints. In particular, each vehicle has to start and end its trip at a central depot and visit a subset of vertices by taking into account, the limitation on the capacity and travel time (length). Some variants of the standard VRP are introduced in the literature. Among them we can refer to the multi depot VRP [2], open VRP [3], VRP with backhauls [4] and the generalized VRP [5,6]. Another variation of the standard VRP is the heterogeneous fixed fleet vehicle routing problem (HFFVRP), in which the vehicles could be different in type. In particular, we are given a fixed number of vehicles of each type and the problem is to assign vertices to different vehicle types to minimize the total length traveled. This problem has many practical applications and comparing to other variations of the VRP has received less attention.

Taillard [7] proposed a column generation heuristic method to solve the HFFVRP. This method is based on adaptive memory procedure (AMP) [8] proposed for the standard VRP, which uses an embedded tabu search (TS) [9]. Particularly, for each type of vehicle, the method solves a succession of homogeneous VRPs resulting in a large number of vehicle routes. Finally, a feasible solution is obtained for the HFFVRP by solving an integer linear programming (ILP) model to optimality, in such way that each variable, column, of the ILP model is a route generated by applying the TS algorithm.

Tarantilis et al. [10,11] proposed two algorithms that incorporate the threshold accepting metaheuristic [12] and simple *swap* plus *extraction-reinsertion* moves to reach high quality solutions. In [10], a set of fixed threshold values are given during the search process while in [11], the idea is to achieve a balance between *diversification* and *intensification* by changing the threshold values dynamically. Computational results show that the second strategy, i.e. dynamically changing the threshold values during the search, is able to produce much better results comparing to the first strategy.

The record-to-record travel algorithm, proposed by Dueck [13] as a deterministic variation of the Simulated Annealing [14], is a general approach to face different optimization problems. Li et al. [15] adapted this algorithm to the HFFVRP.

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According on this method, uphill moves are allowed to escape from local optimum. In other words, the new solution S', obtained from S by doing a series of local searches, could be accepted as a current solution for doing further local searches, whenever, its corresponding objective function is within a given percentage of the objective function of S.

Li et al. [16] proposed a multi-start adaptive memory programming for the HFFVRP. In their method, the adaptive memory procedure applies a heuristic method to generate vehicle routes. Following this step, a modified TS algorithm is designed to improve the quality of the solutions. In a multi-start fashion, this process is repeated for a given number of iterations. Finally, a post optimization procedure, which applies a series of 2-Opt and 3-Opt moves, is applied to the best found solution. Moreover, the path-relinking metaheuristic [17] is used, to enhance the performance of the proposed algorithm [16].

Very recently, a TS metaheuristic has been developed by Brandão to solve the HFFVRP [18]. In this method, an involved algorithm has been proposed based on all of the main TS components, which in some instances lead to further improvements in the previous best known results.

Many papers have investigated the heterogeneous fleet vehicle routing problem (HFVRP). In this case, the number of vehicles of each type is supposed to be unlimited. Since this variation is not included in the scope of this paper, we just refer the readers to the papers [19–21] for more details.

To the best of our knowledge, nobody else has worked on the HFFVRP, so far. The ideas behind the proposed methods, reviewed in this section, are based on heuristic or metaheuristic approaches. Generally, the most studied methods for different optimization problems, rely on exact or heuristic ideas while the combination of these two approaches has received much less attention. In this paper, we suggest a general method which is based on ILP and heuristic techniques to improve the quality of a given initial solution. This method, described later in Section (3), is very general and could be adapted to some other variants of routing problems.

The paper is organized as follows: Section (2) gives a formal description of the HFFVRP. The details of our proposed method are provided in Section (3) followed by computational results in Section (4). Finally, conclusions and future directions are drawn in Section (5).

2. Problem statement

Given a directed graph $G = (N \cup \{0\}, A)$ in which $N = \{1, 2, \ldots, n\}$ is the set of vertices and $\{0\}$ represents the depot. Moreover, $A = \{(i,j); i,j \in N \cup \{0\}\}$ is the set of arcs and the set of vehicle types is represented by $V = \{1, 2, \ldots, m\}$. Also, we assume that the maximum number of each vehicle type $v \in V$ is given in advance, M_v, while each having a capacity Q_v. Moreover, to each vehicle type $v \in V$ is associated a fixed cost f_v. The assumption is that each vertex i has a demand d_i which has to be supplied by exactly one vehicle.

The variables to model the problem are as follows:

$$x_{ijv} = \begin{cases} 1 & \text{if arc } (i,j) \text{ is traversed by vehicle } v \text{ in the optimal solution}, \\ 0 & \text{otherwise}. \end{cases} \tag{1}$$

Also, let us define y_{ij}, as the load of a vehicle after leaving vertex i and before arriving to vertex j. In the definition of y_{ij}, we assume that vertices i nad j, are visited consecutively along arc (i,j).

The model reads as follows:

$$\min \quad \sum_{i \in N \cup \{0\}} \sum_{j \in N \cup \{0\}} \sum_{v \in V} c_{ijv} x_{ijv} + \sum_{v \in V} f_v \sum_{j \in N} x_{0jv} \tag{2}$$

$$\text{subject to}: \quad \sum_{v \in V} \sum_{i \in N} x_{ijv} = 1 \quad \forall j \in N, \tag{3}$$

$$\sum_{i \in N \cup \{0\}} x_{ijv} - \sum_{k \in N \cup \{0\}} x_{jkv} = 0 \quad \forall j \in N, v \in V, \tag{4}$$

$$\sum_{j \in N} x_{0jv} \leqslant M_v \quad \forall v \in V, \tag{5}$$

$$\sum_{i \in N \cup \{0\}} y_{ij} - \sum_{i \in N \cup \{0\}} y_{ji} = d_j \quad \forall j \in N, \tag{6}$$

$$d_j x_{ijv} \leqslant y_{ij} \leqslant (Q_v - d_i) x_{ijv} \quad \forall i,j \in N \cup \{0\}, v \in V, \tag{7}$$

$$x_{ijv} \in \{0,1\} \quad \forall i,j \in N \cup \{0\}, \forall v \in V, \tag{8}$$

$$y_{ij} \geqslant 0 \quad \forall i,j \in N \cup \{0\}. \tag{9}$$

The objective function (2) is to minimize the total costs consisting of the routing and the fixed cost concerning the use of different vehicle types to serve the vertices. Constraints set (3) make sure that each vertex has to be visited exactly once, while constraints set (4) make sure that the same vehicle has to visit and leave each vertex of the solution. Constraints set (5) is a set of forcing constraints which impose the model to use at most a given number of vehicles of each type. Constraints (6) are commodity flow constraints. Particularly, they ensure that the difference between the load of a vehicle before and after visiting a vertex j, is equal to the demand of vertex j. Constraints set (7) are capacity constraints for each type of vehicles, used in the solution. Finally, sets (8) and (9) are decision variables. For a complete survey on different models proposed for other variants of the HVRP, we refer the readers to the papers [22–24].