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Effects of prey refuge on a ratio-dependent predator–prey model with stage-structure of prey population

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ABSTRACT

In this paper, a stage-structured predator–prey model is proposed and analyzed to study how the type of refuges used by prey population influences the dynamic behavior of the model. Two types of refuges: those that protect a fixed number of prey and those that protect a constant proportion of prey are considered. Mathematical analyses with regard to positivity, boundedness, equilibria and their stabilities, and bifurcation are carried out. Persistence condition which brings out the useful relationship between prey refuge parameter and maturation time delay is established. Comparing the conclusions obtained from analyzing properties of two types of refuges using by prey, we observe that value of maturation time at which the prey population and hence predator population go extinct is greater in case of refuges which protect a constant proportion of prey.

1. Introduction

Refuge use is any strategy that decreases predation risk. The existence of refuges can clearly have important effects on the co-existence of predators and prey. In the existing literature, two types of refuges: those that protect a constant number of prey and those that protect a constant proportion of prey have been considered [1]. McNair [2], Sih [3], Ruxton [4] and Scheffer and de Boer [5] etc. proposed and analyzed predator–prey models incorporating prey refuges. Ruxton [4] proposed a continuous-time predator–prey model taking into consideration that the rate at which prey move to the refuge is proportional to predator density. Krivan [6] investigated the influence of refuges used by prey on the dynamics of predator–prey system in two patch environment, where one patch is the refuge for prey while the other is an open habitat. The results showed that optimal antipredator behavior of prey leads to the persistence and reduction of oscillations in population densities. Taking a cue from this, Gonzalez-Olivars and Ramos-Jiliberto [7], Kar [8] and Haung et al. [9] derived a predator–prey model with Holling type III functional response incorporating prey refuges and evaluated the effects with regard to the stability of the interior equilibrium. They obtained that the refuges used by prey can increase the stability of the interior equilibrium. Ma et al. [10] studied the effects of two types of refuges used by prey on a predator–prey interaction with a class of functional responses. After analyzing the stability properties of two types of refuges using by prey, they noted that refuges which protect a constant number of prey have stronger stabilizing effect on the dynamics than the refuges which protect a constant proportion of prey, which is agreement with previous works [6,7]. Chen et al. [11] discussed the instability and global stability properties of the equilibria and the existence and uniqueness of limit cycle of a predator–prey model with Holling type II functional response incorporating a prey refuge. They found that dynamic behavior of the model very much depends on the prey refuge parameter and increasing amount of refuge could increase prey densities and lead to predator outbreaks.

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In the natural world, there are many species (particularly mammalian population) whose individual members have a life history that takes them through two stages: immature and mature with a time lag. Aiello and Freedman [12], Aiello et al. [13] and Song and Chen [14] proposed and analyzed single species growth models with stage structure consisting of immature and mature stages. Agarwal and Devi [15] studied the effects of toxicants on the dynamics of a single species growth model with stage structure consisting of immature and mature stages. They showed that equilibrium level of immature population is more prone for the effects of toxicants in comparison to the mature population. Wang and Chen [16] and Song and Chen [17] proposed and analyzed predator–prey models with stage structure. Shi and Chen [18] studied a ratio-dependent predator–prey model with stage structure in the prey and obtained sufficient conditions for the existence and stability of all equilibria. Agarwal and Devi [19] proposed and analyzed a ratio-dependent predator–prey model where the prey population is stage-structured and the predator population is influenced by the resource biomass. They noted that the influence of resource biomass on the predator population may lead to the extinction of prey population at a lesser value of maturity time in comparison to the absence of the resource biomass. However, as for as our knowledge goes, we note that effects refuges using by prey on the dynamics of stage-structured predator–prey system have not been considered.

In this paper, a model describing the ratio-dependent predator–prey interaction with stage structure of prey population consisting of immature and mature stages with constant time of maturation delay is proposed. The effects of refuges used by mature prey population on the dynamics of considered model is investigated. This paper is concerned with questions of stability, bifurcation and persistence of populations. But, the main question of this paper is: How prey refuges affect the critical value of maturation time (the value of maturation time at which stability change occurs). The stability theory of delay differential equations is used to analyze the model. To accomplish our analytical findings and to observe the effects of important parameters on the dynamics of the system, computer simulations are carried out.

2. The mathematical model

In this paper, we consider a model given by the set of following differential equations:

$$\begin{aligned}
\dot{x}_i(t) &= \alpha x_m(t) - \gamma x_i(t) - \alpha e^{-\gamma\tau} x_m(t-\tau),\\
\dot{x}_m(t) &= \alpha e^{-\gamma\tau} x_m(t-\tau) - \beta x_m^2(t) - \frac{(x_m(t) - x_{mr}(t))y(t)}{(x_m(t) - x_{mr}(t)) + y(t)},\\
\dot{y}(t) &= \frac{k(x_m(t) - x_{mr}(t))y(t)}{(x_m(t) - x_{mr}(t)) + y(t)} - dy(t),
\end{aligned} \tag{2.1}$$

where $x_i(t)$ and $x_m(t)$ represent the densities of immature and mature prey populations, respectively at time t. $y(t)$ is the density of predator population at a time t.

Model (2.1) is derived under the following assumptions:

(H1): The prey population: we first assume that the prey population is divided in two stages: immature and mature. $\alpha > 0$; the birth rate of the immature population, $\gamma > 0$; the death rate of immature population , $\beta > 0$; the death rate of the mature population, $\tau > 0$; the length of time from birth to maturity. $e^{-\gamma\tau}$ denotes the surviving rate of immature stage to reach maturity. The term $\alpha e^{-\gamma\tau} x_m(t-\tau)$ represents the immature prey individuals who were born at time $(t-\tau)$ and still survive at time t, and represents the transformation of immature population to mature population.

(H2): Growth rate of predator population is wholly dependent on prey population and it is assumed that the predators feed only on the mature prey population because for a number of animals immature prey population concealed in the mountain cave depends on their parents. $k > 0$ is the efficiency with which predators convert consumed prey into new predators. $d > 0$ is the death rate of predators.

(H3): The quantity x_{mr} is considered because of two alternative points of view: (i) $x_{mr} = p$, where $p > 0$ denotes the fixed quantity of mature prey. (ii) $x_{mr} = \zeta x_m$, the quantity of mature prey population using refuges is proportional to the existing mature prey population with a proportionality constant $0 < \zeta < 1$.

2.1. Case when a constant number of prey using refuges

When a fixed quantity of prey using refuges, the model (2.1) takes the following form:

$$\begin{aligned}
\dot{x}_i(t) &= \alpha x_m(t) - \gamma x_i(t) - \alpha e^{-\gamma\tau} x_m(t-\tau),\\
\dot{x}_m(t) &= \alpha e^{-\gamma\tau} x_m(t-\tau) - \beta x_m^2(t) - \frac{(x_m(t) - p)y(t)}{(x_m(t) - p) + y(t)},\\
\dot{y}(t) &= \frac{k(x_m(t) - p)y(t)}{(x_m(t) - p) + y(t)} - dy(t),\\
x_m(t) &= \phi_m(t) \geqslant 0, \ -\tau \leqslant t < 0 \text{ and } x_i(0) > 0, \ x_m(0) > p, \ y(0) > 0.
\end{aligned} \tag{2.2}$$