Applied Mathematical Modelling 35 (2011) 3396–3407

Contents lists available at ScienceDirect

Applied Mathematical Modelling

journal homepage: www.elsevier.com/locate/apm

Mathematical modelling and analysis of bulk waves in rotating generalized thermoelastic media with voids

J.N. Sharma [a,*], D. Grover [a], D. Kaur [b]

[a] Department of Mathematics, National Institute of Technology, Hamirpur 177005, India
[b] Department of Mathematics, Dr. B. R. Ambedkar National Institute of Technology, Jalandhar 144011, India

ARTICLE INFO

Article history:
Received 7 May 2009
Received in revised form 6 January 2011
Accepted 11 January 2011
Available online 18 January 2011

Keywords:
Thermal relaxation
Volume fraction
Kibel number
Thermoelastic waves
t-test
Rotation

ABSTRACT

The present paper deals with the propagation of body waves in a homogenous isotropic, rotating, generalized thermoelastic solid with voids. The complex quartic secular equation has been solved by employing Descartes' algorithm and perturbation method to obtain phase velocities, attenuations and specific loss factors of four attenuating and dispersive waves, which are possible to exist in such media. These wave characteristics have been computed numerically for magnesium crystal and presented graphically. Statistical analysis has been performed to compare the obtained computer simulated result in order to have estimate on the suitability of the method to compute various characteristics of the waves. This work may find applications in geophysics and gyroscopic sensors.

1. Introduction

The theory of elastic materials with voids is concerned with the elastic materials consisting of a distribution of small pores (voids) which contains nothing of mechanical or energetic significance. Goodman and Cowin [1] established a general continuum theory for a fluid like material with voids and applied it to flow of granular materials. The nonlinear theory of elastic materials with voids was proposed by Nunziato and Cowin [2] and the linearized version was deduced by Cowin and Nunziato [3] where voids have been included as an additional kinematics' variable. This theory reduces to the classical theory of elasticity in the limiting case when void volume vanishes. Puri and Cowin [4] presented a complete analysis of the frequency equation for plane waves in a linear elastic material with voids and observed that there are two dispersive and attenuated dilatational waves in this theory, one is predominantly the dilatational wave of classical linear elasticity and other is predominantly a wave carrying a change in void volume fraction. Chandrasekharaiah [5] established the uniqueness of solution of an initial boundary value problem formulated completely in terms of stress and volume fraction fields for homogenous and isotropic materials with voids. Chandrasekharaiah [6] studied the propagation of plane waves in a homogenous and isotropic unbounded elastic solid with voids rotating with a uniform angular velocity.

Moreover, the effect of voids on surface waves propagating in thermoelastic media has got its due importance where the situation so demands. The cooling and heating of the medium also results in the expansion and contraction of the voids along with the core material which contributes towards thermal stress and vibration developments in solids. Sharma and Kaur [7] studied the plane harmonic waves in generalized thermoelastic materials with voids and Sharma et al. [8] studied an exact free vibration analysis of simply supported, homogenous isotropic, cylindrical panel with voids in the three dimensional

* Corresponding author. Tel.: +91 1972 254122; fax: +91 1972 223834.
E-mail addresses: jns@nitham.ac.in (J.N. Sharma), groverd2009@gmail.com (D. Grover), kaurd@nitj.ac.in (D. Kaur).

0307-904X/$ - see front matter
doi:10.1016/j.apm.2011.01.014

generalized thermoelasticity. Singh and Tomar [9] explored the possibility of plane wave propagation in an infinite thermoelastic medium with voids using the theory developed by Iesan and investigated the reflection phenomenon of a set of coupled longitudinal waves from a free plane boundary of a thermoelastic half-space with voids.

Auriault [10] and Sharma and Grover [11] studied the body wave propagation in an infinite homogenous isotropic elastic and thermoelastic media respectively rotating with uniform angular velocity. During the last three decades, non-classical theories, Lord and Shulman [12], Green and Lindsay [13] have been developing to alleviate the paradox inherited in the classical theory of heat conduction which predicts an infinite speed of heat transportation and contradicts the physical facts. Chandrasekharaiah [14] referred a wave-like thermal disturbance as 'second sound'. Ackerman and Overtone [15] and Ackerman et al. [16] have supported the experimental exhibition of the actual occurrence of 'second sound' at low temperatures and small intervals of time in solid helium.

In the present study, wave propagation in a homogenous isotropic, thermoelastic media with voids which is rotating with uniform angular rotation $\vec{\Omega}$, about a fixed axis $\vec{e}_3$, the wave propagation of a perturbation displacement in the plane $(\vec{e}_1,\ \vec{e}_2)$ perpendicular to $\vec{e}_3$ may be affected by the Coriolis force. The wave propagation is investigated as a function of the Kibel number $\Gamma = \frac{\omega}{\Omega}$. The quartic complex polynomial characteristic equation yields us four complex roots, in general. These roots can be associated with four dispersive waves namely quasi-longitudinal (QL), quasi-transverse (QT), volume fractional (ϕ-mode) and thermal wave (T-mode). The general complex characteristics equation has been solved by using Descartes' algorithm along with irreducible case of Cardano's and series (perturbation) expansion methods with the help of DeMoivre's theorem in order to obtain phase speeds, attenuation coefficients and specific loss factors of the waves. The numerical solution of the secular equations is carried out for magnesium crystal like material in order to illustrate the analytical developments. The computer simulated results have also been presented graphically for velocities, attenuation and phase velocity with respect to Kibel number.

2. Wave equations in rotating media

We consider a homogenous isotropic, thermoelastic solid with voids at uniform temperature T_0 and initial volume fraction ϕ_0 in the undisturbed state. The medium is assumed to be rotating with uniform angular velocity $\vec{\Omega}$. with respect to an inertial frame. The Coriolis effect is caused by the rotation of an object and the inertia of mass experiencing the effect. When Newton's laws are transformed to a rotating frame of reference, the Coriolis and Centrifugal forces appear. Both forces are proportional to the mass of the object. The Coriolis force is proportional to the rotation rate. The Coriolis force acts in a direction perpendicular to the rotation axis and to the velocity of the body in the rotating frame and is proportional to the object's speed in rotating frame. This force is termed as inertial force (pseudo force). The Coriolis force is quite small, only the horizontal component of Coriolis force is generally important.

The basic governing dynamical equations of linear generalized thermoelastic interactions after including the Coriolis and centripetal forces, in absence of body forces and heat sources, Sharma and Kaur [7] and Sharma et al. [8] are

$$\mu\nabla^2\vec{u} + (\lambda+\mu)\nabla\nabla\cdot\vec{u} + b\nabla\phi - \beta\nabla T = \rho\left(\ddot{\vec{u}} + \vec{\Omega}\times\left(\vec{\Omega}\times\vec{u}\right) + 2\left(\vec{\Omega}\times\dot{\vec{u}}\right)\right), \tag{1}$$

$$\alpha\nabla^2\phi - \xi_1\phi - \xi_2\dot{\phi} - b\nabla\cdot\vec{u} + mT = \rho\chi\ddot{\phi}, \tag{2}$$

$$K\nabla^2 T - \rho C_e\left(\dot{T} + t_0\ddot{T}\right) = \beta T_0\nabla\cdot\left(\dot{\vec{u}} + t_0\ddot{\vec{u}}\right) + mT_0\left(\dot{\phi} + t_0\ddot{\phi}\right), \tag{3}$$

where t_0 is the thermal relaxation time, $\vec{u}(x_1,x_2,x_3,t) = (u_1,u_2,u_3)$ is the displacement vector; $T(x_1,x_2,x_3,t)$ is the temperature change; λ, μ are Lamé's parameters; K is thermal conductivity; ρ and C_e are respectively, the density and specific heat at constant strain; ϕ is change in Volume fraction, $\beta = (3\lambda + 2\mu)\alpha_T$, α_T is the linear thermal expansion. α, b, ξ_1, ξ_2, m and χ are material constants due to the presence of voids. Here superposed dot represents time differentiation.

We define the non-dimensional quantities:

$$\begin{aligned}
&x_i' = \frac{\omega^* x_i}{c_1},\quad t' = \omega^* t,\quad u_i' = \frac{\rho\omega^* c_1 u_i}{\beta T_0},\quad \phi' = \frac{\omega^{*2}\chi\phi}{c_1^2},\quad T' = \frac{T}{T_0},\quad \varepsilon_T = \frac{\beta^2 T_0}{\rho C_e(\lambda+2\mu)},\quad \delta_1^2 = \frac{c_3^2}{c_1^2},\\
&t_0' = \omega^* t_0,\quad \Omega' = \frac{\Omega}{\omega^*},\quad \delta^2 = \frac{c_2^2}{c_1^2},\quad a_1 = \frac{bc_1^2}{\beta T_0\chi\omega^{*2}},\quad a_2 = \frac{b\chi\beta T_0}{\alpha\rho c_1^2},\quad a_3 = \frac{\xi_1 c_1^2}{\alpha\omega^{*2}},\quad a_4 = \frac{mT_0\chi}{\alpha},\\
&a_5 = \frac{mc_1^4}{K\chi\omega^{*3}},\quad \bar{\xi} = \frac{\xi_2\omega^*}{\xi_1},\quad k' = \frac{c_1 k}{\omega^*},\quad \omega' = \frac{\omega}{\omega^*},\quad c' = \frac{c}{c_1},
\end{aligned} \tag{4}$$

where

$$\omega^* = \frac{C_e(\lambda+2\mu)}{K},\quad c_1^2 = \frac{\lambda+2\mu}{\rho},\quad c_2^2 = \frac{\mu}{\rho},\quad c_3^2 = \frac{\alpha}{\rho\chi}$$

Upon using quantities (4) in Eqs. (1)–(3), we obtain

$$\delta^2\nabla^2\vec{u} + (1-\delta^2)\nabla(\nabla\cdot\vec{u}) + a_1\nabla\phi - \nabla T = \ddot{\vec{u}} + \vec{\Omega}\times\left(\vec{\Omega}\times\vec{u}\right) + 2\left(\vec{\Omega}\times\dot{\vec{u}}\right), \tag{5}$$