Applied Mathematical Modelling 37 (2013) 7708–7716

Contents lists available at SciVerse ScienceDirect

Applied Mathematical Modelling

journal homepage: www.elsevier.com/locate/apm

Simulation and optimization for crop water allocation based on crop water production functions and climate factor under uncertainty

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ARTICLE INFO

Article history:
Received 1 May 2012
Received in revised form 4 November 2012
Accepted 1 March 2013
Available online 15 March 2013

Keywords:
Crop water production function
Meteorological factor
Optimal allocation
Water resources
Irrigation
Uncertainty

ABSTRACT

In this paper, the uncertainty methods of interval and functional interval are introduced in the research of the uncertainty of crop water production function itself and optimal allocation of water resources in the irrigation area. The crop water production functions in the whole growth period under uncertainty and the optimal allocation of water resources model in the irrigation area under uncertainty are established, and the meteorological factor is considered in the model. It can promote the practical application of the uncertain methods, reflect the complexity and uncertainty of the actual situation, and provide more reliable scientific basis for using water resources economically, fully improving irrigation efficiency, and keeping the sustainable development of the irrigated area. This approach has important value on theoretical and practical for the optimal irrigation schedule, and has very broad prospects for research and development to other related agriculture water management.

1. Introduction

The increased water deficits associated with overuse of surface water and the declining groundwater levels are threatening the sustainability of agricultural production in the regions of Northwest of China [1]. Therefore, the improvement of water management practices is the utmost importance to ensure crop production with existing supplies of water [2,3]. Crop water production function is the mathematical description of the influence of water supply time and quantity on crop yield during crop growth. It is the main basis of evaluating various irrigation strategies and the foundation of determining economic water quotas and optimal allocation of water.

In order to use limited water resources reasonably to achieve maximum crop yield and schedule the reasonable allocation of limited water supply both in temporal and spatial scale, the research on crop water production function was rapidly developed [4–8]. For example, De Juan et al. [9] developed a model for optimal cropping patterns within the farm based on crop water production functions and irrigation uniformity. Igbadun et al. [10] presented the performance evaluation of four selected crop water production functions. Brumbelow and Georgakakos [11] presented a new algorithm type based upon differential crop yield response to irrigation that uses these crop models to determine planning-level irrigation schedules and crop water production functions.

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0307-904X/$ - see front matter
http://dx.doi.org/10.1016/j.apm.2013.03.018

In previous research on crop water production function, the problem was simplified into the deterministic problem, and only the deterministic function of the relationship between crop yield and evapotranspiration was established, so evapotranspiration corresponding to the crop highest yield also was the deterministic value. However, there are a number of uncertain factors in the actual problems, so the actual application value of the research results is reduced, due to the uncertain factors did not be considered in reflecting the practical problems. At present, generally the deterministic optimal allocation of water resources in the irrigation area is studied [12]. Some scholars consider the random factors (e.g. rainfall, runoff) and establish some stochastic programming models [13,14]. However, still no scholar has researched the uncertainty of water production function itself and its application on the optimization of irrigation water distribution systematically.

Climate change has the impacts on many fields. There are a number of studies have been undertaken in the last decades to evaluate the impacts of climate change on water resources and agriculture [15–19]. However, there are few studies to consider the meteorological factor in optimal allocation of water resources in the irrigation area, especially in optimal allocation of water resources in the irrigation area under uncertainty.

In this paper, the uncertainty methods of interval and functional interval are introduced in the research of crop water production function and optimal allocation of water resources in the irrigation area to accurately reflect the real-world problem. The crop water production functions in the whole growth period under uncertainty are firstly presented. Then based on above functions, the optimal allocation of water resources model in the irrigation area under uncertainty is established, and the meteorological factor is introduced in the model. It can promote the practical application of the uncertainty methods and provide more reliable scientific basis for using water resources economically, fully improving irrigation efficiency and keeping the sustainable development of the irrigated area. This approach has important value on theoretical and practical for the optimal irrigation schedule, and has very broad prospects for research and development to other related agriculture water management.

2. Study area

The research area is located in Minqin, Gansu Province, China. It belongs to Wuwei region (101°49′∼104°12′E, 38°03′∼39°28′N) and the central area is the Shiyang River of alluvial narrow and flat oasis belt. It has the characteristic of North low and South high terrain, 1200–1500 m above sea level. The natural gradient is about l per thousand. It is surrounded by high terrain with basin landform characteristics [20]. In recent decades, due to the impact of human activities and climate change, severe land desertification appeared in Minqin. The ecological environment is deteriorating and water shortage has been very prominent to the front.

Hongyashan irrigation region locates in the oasis belt of Minqin County. Its effective irrigation area is about 63073.3 hectare, and it is a typical continental temperate arid climate. Annual precipitation is less than 150 mm, while annual evaporation capacity is 2000–2600 mm. Irrigation water resources available are mainly from hongyashan reservoir water and groundwater. Due to excessive use in the middle irrigation area, surface water decreased year by year, and to 2003 has been less than 100 million m^3. Groundwater which was not repeated with the surface water had only 31 million m^3. Water resources is very scarce, and agriculture implements deficient irrigation [21].

3. Crop water production function in the whole growth period under uncertainty

3.1. Method of Interval regression analysis

The selected research crops, spring wheat, cotton, seed melon and honey dew melon are main crops planted in Minqin. The data of crop water consumption in the whole growth period and yield came from pilot study on water production functions and optimal irrigation schedule of main crops in Shiyang river basin [22].

Due to the information used for the water production functions are not the same, the interval regression analysis is useful to address the collected information. An interval linear regression model can be written as

$$Y(x) = A_0 + A_1x_1 + \cdots + A_nx_n = Ax$$

Where $x = (1, x_1, \ldots, x_n)^t$ is a real input vector, $A = (A_0, \ldots, A_n)$ is an interval coefficient vector, and $Y(x)$ is the corresponding estimated interval. An interval coefficient A_i is denoted as $A_i = (a_i, c_i)$ where a_i is a center and c_i is a radius.

There are many approaches to achieve the interval regression analysis, such as translating into quadratic optimization problems, using neural networks, using support vector machines. This research used the relatively mature interval regression analysis method based on quadratic programming, and integrates the central tendency of least squares and possibilistic property of fuzzy regression. The proposed model can be represented as follows [23]:

$$\underset{a,c}{Min} J = k_1\sum_{j=1}^{p}(y_j - a^t x_j)^2 + k_2\sum_{j=1}^{p}c^t|x_j||x_j|^t c$$

Subject to $a^t x_j + c^t|x_j| \geqslant y_j$,

$$a^t x_j - c^t|x_j| \leqslant y_j, \quad j = 1, \ldots, p$$