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Integrated mathematical forms for update of physical parameter matrices of damped dynamic system

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ABSTRACT

Mathematical modelling and updating of damped dynamic systems that involve some modelling errors and subsequent analysis based on those errors will lead to inaccuracy in the results. Because measured and analytical data are unlikely to be identical due to measurement noise and model inadequacies, it is necessary to estimate more accurate parameter matrices for design and analysis. By minimizing a cost function expressed as the sum of the norms of the difference between analytical and experimental parameter matrices, this study directly derives the integrated mathematical expressions for updated physical parameter matrices. In the derivation process, the eigenfunction of a damped dynamic system is utilized as a constraint equation for the updating. It is illustrated that the proposed methods take more explicit forms and can be widely utilized in the damped and undamped systems. Based on the comparison with other methods, the validity of the proposed methods is demonstrated in numerical applications.

1. Introduction

Finite element modelling of real structural or mechanical systems should be well formulated such that it can accurately predict the dynamic and static characteristics of the system under any disturbance. Numerically simulated results using finite element models can deviate from the actual responses due to modelling errors and differences between the measured and analytical data. Thus, any subsequent analysis based on these inaccuracies may be flawed without any modelling correction.

Boundary conditions, geometry, and material properties are the major parameters that can have significant effects on the responses predicted by the finite element model. These parameters are subjected to uncertainties, which lead to errors in the modelling process. Updating the physical parameter matrices for proper simulation and design studies concerns the correction of the finite element model. The finite element model should be updated using the data measured from the real systems, although the possibility of measurement errors should not be totally ruled out.

Modal data such as natural frequencies and corresponding mode shapes are extremely useful information that can assist in the design and analysis of almost any structure. The development of a modal model and visualization of mode shapes in the design process can assist in the simulation and design studies. The measured data obtained from modal tests do not usually satisfy the dynamic equation derived by the finite element model. Thus, in order to overcome the discrepancies, the model should be refined or updated using the modal data measured from the real systems.

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Assuming the dynamic system of a Rayleigh damping, two approaches can be considered to predict the physical parameter of the mass and stiffness matrices. One approach is to modify the analytical mass matrix to satisfy the orthogonality condition based on the measured modal data, and subsequently modify the stiffness matrix to fulfill the eigenvalue equation as a function of the measured mode shapes, natural frequencies, and corrected mass matrix. An alternative approach is to firstly correct the stiffness matrix and then modify the analytical mass matrix to fulfill the eigenvalue equation based on the modal test data and the updated stiffness matrix. In both approaches, the damping matrix is estimated from the modal test data, the updated mass and stiffness matrices, and the satisfaction of the eigenfunction constraint.

There have been many analytical methods for correcting analytical models to predict test results more closely. Friswell and Mottershead [1,2] provide a comprehensive overview that illustrates many of the different techniques, such as the direct updating methods, parameter updating methods, and model updating methods. Baruch [3,4] proposed a method to correct the stiffness matrix based on the measured mode shapes obtained from vibration tests, by minimizing a cost function to use the positive definite symmetric mass matrix as the weighting matrix. Berman [5] described the required changes in the mass matrix to satisfy the orthogonality relationship using a minimum-weighted Euclidean norm and the method of Lagrange multipliers. Berman and Nagy [6] proposed a direct method to identify a set of minimum changes in the analytical matrices that force the eigensolutions to agree with the test measurements. Kabe [7] adjusted the stiffness matrix such that the percentage change to each stiffness coefficient is minimized. The method preserved the physical configuration of the analytical model and reproduced the modes used in the identification. Caeser and Pete [8] suggested two updating strategies that involve either updating the mass matrix followed by updating the stiffness matrix or updating the stiffness matrix followed by updating the matrix. The authors discussed two methods for the direct updating of mathematical models based on modal test data. Based on the development of a symmetric eigenstructure assignment algorithm, Zimmerman and Widengren [9] determined the residual damping and stiffness matrices such that the improved analytical model eigenstructure matches more closely that obtained from experiments. Using the element correction method combined with the Lagrange multiplier technique, Wei [10] proposed an analytical method to correct and modify both analytical mass and stiffness matrices simultaneously using the vibration test data. Utilizing the fundamental linear algebra such as the Moore–Penrose inverse, Lee and Eun [11,12] derived explicit forms of the parameter matrices by minimizing performance indices in the satisfaction of the eigenfunction and orthogonality requirements. Yang and Chen [13] presented a method to update the mass and stiffness matrices of a structural model to reproduce the frequencies measured from the structure. Lin and Zhu [14] developed model updating methods to identify mass, stiffness, and damping matrices of damaged dynamic systems based on the frequency response function (FRF) method. Reix et al. [15] presented an analytical method to update the damping matrix using FRF sensitivity. Based on the localization of the modelling errors, Liu and Yuan [16] provided methods to update the damping and stiffness matrices of damped systems with localized modelling errors.

This study considers the update of physical parameter matrices of damped dynamic systems. They are derived by minimizing a cost function expressed as the sum of the norms of the difference between the updated and analytical matrices. The eigenfunction of the damped system was utilized as a constraint condition in the updating process. Using the fundamental linear algebra without any multiplier method, the integrated mathematical forms of the stiffness, mass, and damping matrices are straightforwardly derived. Based on the comparison with other methods, whether the proposed methods take more explicit forms and can be widely utilized in the damped and undamped systems is investigated. The validity of the proposed methods is illustrated in numerical applications.

2. Derivation of updated parameter matrices

Using the finite element method, the dynamic equation of the free motion of a damped dynamic system is expressed as a system of a second-order differential equation. The dynamic response of a structure that is assumed to be linear and approximately discretized for n degrees of freedom without external excitation can be described by the equations of motion

$$\mathbf{M}_a\ddot{\mathbf{u}} + \mathbf{C}_a\dot{\mathbf{u}} + \mathbf{K}_a\mathbf{u} = \mathbf{0}, \tag{1}$$

where $\mathbf{M}_a$, $\mathbf{C}_a$ and $\mathbf{K}_a$ denote the $n \times n$ analytical mass, damping, and stiffness matrices, $u = [\,u_1 \quad u_2 \quad \cdots \quad u_n\,]^T$.

Assume the solution of the displacement vector of Eq. (1) in exponential form of

$$\mathbf{u} = \psi_a e^{\lambda_a t}, \tag{2}$$

where ψ_a denotes the modal coordinate vector and λ_a is the natural frequency. The substitution of Eq. (2) into Eq. (1) with $e^{\lambda_a t} \neq 0$ yields the equation

$$\mathbf{M}_a\boldsymbol{\varphi}_a\Lambda_a^2 + \mathbf{C}_a\boldsymbol{\varphi}_a\Lambda_a + \mathbf{K}_a\boldsymbol{\varphi}_a = \mathbf{0}, \tag{3}$$

where $\Lambda_a = diag([\,\lambda_{a,1} \quad \lambda_{a,2} \quad \cdots \quad \lambda_{a,m}\,])$, $\Lambda_a^2 = diag\left(\left[\,\lambda_{a,1}^2 \quad \lambda_{a,2}^2 \quad \cdots \quad \lambda_{a,m}^2\,\right]\right)\boldsymbol{\varphi}_a$ denotes the $m \times n$ eigenvector matrix corresponding to the first m eigenvalues ($m < n$).

Assume the measured natural frequencies and the corresponding mode shapes of the real system as Λ and $\boldsymbol{\varphi}$, respectively. Then the eigenfunction is replaced by

$$\mathbf{M}\boldsymbol{\varphi}\Lambda^2 + \mathbf{C}\boldsymbol{\varphi}\Lambda + \mathbf{K}\boldsymbol{\varphi} = \mathbf{0}, \tag{4}$$