Applied Mathematical Modelling 35 (2011) 1222–1230

Contents lists available at ScienceDirect

Applied Mathematical Modelling

journal homepage: www.elsevier.com/locate/apm

An optimal repair–replacement policy for a cold standby system with use priority ☆

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ARTICLE INFO

Article history:
Received 1 May 2009
Received in revised form 4 August 2010
Accepted 10 August 2010
Available online 19 August 2010

Keywords:
Geometric process
Renewal process
Priority
Replacement policy
Renewal reward theorem
Convolution

ABSTRACT

In this paper, the maintenance problem for a cold standby system consisting of two dissimilar components and one repairman is studied. Assume that both component 1 and component 2 after repair follow geometric process repair and component 1 is given priority in use when both components are workable. Under these assumptions, using geometric process repair model, we consider a replacement policy N under which the system is replaced when the number of failures of component 1 reaches N. Our purpose is to determine an optimal replacement policy N^* such that the average cost rate (i.e. the long-run average cost per unit time) of the system is minimized. The explicit expression for the average cost rate of the system is derived and the corresponding optimal replacement policy N^* can be determined analytically or numerically. Finally, a numerical example is given to illustrate some theoretical results and the model applicability.

1. Introduction

In the earlier study of the repair–replacement problem, the repair–replacement models are mainly concentrated on the study of the model for a simple repairable system (i.e. a one-component repairable system with one repairman). Moreover, it is usually assumed that the system after repair is "as good as new". This is a perfect repair model. However, this assumption is not always true. In practice, most repairable systems are deteriorative because of the ageing effect and the accumulative wear. Barlow and Hunter [1] first presented a minimal repair model in which a system after repair has the same failure rate and the same effective age as at the time of failure. Brown and Proschan [2], combined the perfect repair and the minimal repair, first reported an imperfect repair model in which the repair will be perfect with probability p or minimal with probability $1 - p$. Research works on the minimal repair model and the imperfect repair model include Park [3], Phelps [4], Block et al. [5], Kijima [6], Martorell et al. [7], Jhang and Sheu [8], Sheu et al. [9], Chiang and Yuan [10], and Jiang and Ji [11].

However, for a deteriorating simple system, it seems is more reasonable to assume that the successive working times of the system after repair will become shorter and shorter, while the consecutive repair times of the system after failure will become longer and longer. Ultimately, it cannot work any longer, neither can it be repaired. For such a stochastic phenomenon, Lam [12,13] first introduced a geometric process repair model to approach it, and under this model, he studied two kinds of replacement policy for simple repairable systems, one based on the working age T of the systems and the other based on the failure number N of the system. The explicit expressions of the average cost rate under these two kinds of policy are respectively calculated, and the corresponding optimal replacement policies T^* and N^* can be found analytically or numerically. Under some mild conditions, he also proved that the optimal policy N^* is better than the optimal policy T^*.

☆ This work was fully supported by the National Natural Science Foundation of China under Grant No. 10801032.
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doi:10.1016/j.apm.2010.08.009

Since, the geometric process is a special monotone process, Stadje and Zuckerman [14] introduced a general monotone process repair model to generalize Lam's work. Zhang [15] combined the two replacement policies used by Lam [12,13] and proposed a bivariate replacement policy (T,N) under which the system is replaced at the working age T or at the time of the Nth failure, whichever occurs first. Under the same conditions in Lam [12,13], he showed that the optimal policy $(T,N)^*$ is better than the optimal policies N^* and T^*. Other research works on the geometric process repair model include Finkelstein [16], Stanley [17], Leung and Lee [18], Leung [19], Zhang et al. [20,21], Zhang [22–24], Wang and Zhang [25], Castro and Pérez-Ocón [26], Chen and Li [27] and others.

In practical application, some complex repairable systems such as series, parallel, standby and k-out-of-n: F (or G) systems and others are often installed. By applying the geometric process repair model, Zhang and Wu [28] first reported some reliability indices of a two-component series repairable system when the operating time of component 1 follows the exponential distribution while that of component 2 and the repair times of both components follow general distributions. Lam and Zhang [29] provided a more in depth analysis of the series system studied by Zhang and Wu [28] under the assumptions that the operating times and repair times of both components all follow the exponential distribution. Further, Zhang and Wang [30] considered a replacement policy $M = (N_1,N_2,\ldots,N_k)$ for a k-dissimilar-component series repairable system, where N_1, N_2, ..., and N_k are, respectively, the number of failures of component 1, component 2, ..., and component k. An optimal replacement policy $M^* = (N_1^*, N_2^*, \ldots, N_k^*)$ can be determined by minimizing the average cost rate of the system. Many research works for some complex repairable systems have been done by Zhang and Lam [31], Zhang et al. [32], Lam and Zhang [33,34], Lam et al. [35] and others along this direction.

Note that the standby redundancy techniques are often used to improve the reliability, raise the availability or reduce the cost of a system in many practical applications. Zhang [36] analyzed a two-component cold standby repairable system with one repairman. Assume that each component after repair is not "as good as new", and obeys a geometric process repair. Under this assumption, he studied a replacement policy N based on the number of repairs of component 1. An optimal replacement policy N^* can be determined by maximizing the long-run expected reward per unit time. Zhang et al. [37] studied two kinds of replacement policy for a two-component cold standby repairable system with one repairman, one based on the working age T of component 1 under which the system is replaced when the working age of component 1 reaches T, and the other based on the failure number N of component 1 under which the system is replaced when the failure number of component 1 reaches N. The optimal replacement policies T^* and N^* are, respectively, determined by minimizing the average cost rate of the system. They also proved that the optimal policy N^* is better than the optimal policy T^* for a cold standby repairable system under some mild conditions.

Similarly, in order to improve the reliability, raise the availability or reduce the cost of a system, the techniques for priority in use or repair are also used. For example, in the operating room of a hospital, an operation must be discontinued if only the power source is cut (i.e. the power station fails). Usually, there is a standby generator (e.g. a storage battery) which can provide electric power when the main power station fails. Thus, the power station (regarded as the main component, written as component 1) and the storage battery (as the cold standby component, written as component 2) form a repairable electricity-supply system. Obviously, it is reasonable to assume that the power station has use priority due to the operating cost of the power station is cheaper than the storage battery. Besides the electricity-supply system in a hospital, some similar examples can be found from Lam [38]. Thus, a two-component cold standby repairable system with one repairman and priority in use or repair is often employed in practical applications. Nakagawa and Osaki [39] assumed that both working time and repair time of the priority component follow general distributions while both working time and repair time of the non-priority component follow exponential distributions, and the repairs are perfect. Under these assumptions, they developed some interesting reliability indices for the system, using the Markov renewal theory. Other similar studies include those by Osaki [40] and Buzacott [41], for generalizations.

In this paper, we apply the geometric process repair model to a two-component cold standby repairable system with one repairman by assuming that each component after repair is not "as good as new" and follows a geometric process repair, and component 1 has use priority. We consider a repair–replacement policy N based on the number of failures of component 1 under which the system is replaced when the failure number of component 1 reaches N. Our purpose is to determine an optimal replacement policy N^* such that the average cost rate of the system is minimized. The explicit expression of the average cost rate of the system is derived and the corresponding optimal replacement policy N^* can be determined analytically or numerically. Finally, a numerical example is given to illustrate some theoretical results included the uniqueness of the optimal replacement policy N^*, the sensitivity analysis, i.e. the influence of the ratio of the geometric process on the optimal solution, and the comparison of the optimal solutions for the model with and without use priority in this paper.

For ease of reference, we first state the definitions of stochastic order and geometric process.

Definition 1. Given two random variables ξ and η, ξ is said to be stochastically larger than η or η is stochastically smaller than ξ, if

$$P(\xi > \alpha) \geqslant P(\eta > \alpha) \quad \text{for all real } \alpha,$$

denoted by $\xi \geqslant_{st} \eta$ or $\eta \leqslant_{st} \xi$ (see, e.g., Ross [42]). Furthermore, it is said that a stochastic process $\{X_n, n = 1,2,\ldots\}$ is stochastically decreasing if $X_n \geqslant_{st} X_{n+1}$ and stochastically increasing if $X_n \leqslant_{st} X_{n+1}$ for all $n = 1,2,\ldots$.