Applied Mathematical Modelling 35 (2011) 1506–1511

Contents lists available at ScienceDirect

Applied Mathematical Modelling

journal homepage: www.elsevier.com/locate/apm

# Single-machine total completion time scheduling with a time-dependent deterioration

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ARTICLE INFO

*Article history:*
Received 23 June 2009
Received in revised form 2 September 2010
Accepted 13 September 2010
Available online 22 September 2010

*Keywords:*
Scheduling
Single-machine
Time-dependent
Deteriorating jobs
Total completion time

ABSTRACT

In this paper, we consider the single-machine scheduling problems with a time-dependent deterioration. By the time-dependent deterioration, we mean that the processing time of a job is defined by an increasing function of total normal processing time of jobs in front of it in the sequence. The objective is to minimize the total completion time. We develop a mixed integer programming formulation for the problem. The complexity status of this problem remains open. Hence, we use the smallest normal processing time (SPT) first rule as a heuristic algorithm for the general cases and analyze its worst-case error bound. Two heuristic algorithms utilize the V-shaped property are also proposed to solve the problem. Computational results are presented to evaluate the performance of the proposed algorithms.


## 1. Introduction

In classical scheduling problems, the processing times of jobs are assumed to be constant values. However, in many/various practical/real life settings/applications, job processing times are an increasing function of their starting times or their positions in the sequence. This phenomenon, known as deterioration, has been extensively studied in the last decade in various machine settings and performance measures. Extensive surveys of research related to scheduling deteriorating jobs can be found in Alidaee and Womer [1] and Cheng et al. [2]. More recent papers which have considered scheduling jobs with deteriorating jobs include Wu et al. [3], Shiau et al. [4], Wang et al. [5], Toksar and Guner [6], Wang and Cheng [7], Wang et al. [8,9], Lee et al. [10], Cheng et al. [11,12], Wu and Lee [13,14], Toksar and Guner [15], Wang et al. [16], Wang et al. [17], Ji and Cheng [18], Ng et al. [19] and Yin et al. [20]. Wu et al. [3] considered single-machine total weighted completion time scheduling problem under linear deterioration. They proposed a branch-and-bound method and several heuristic algorithms to solve the problem. Shiau et al. [4] considered two-machine flowshop scheduling to minimize mean flow time with simple linear deterioration. Wang et al. [5] considered the single-machine scheduling problems with deterioration jobs and group technology assumption. They showed that the makespan minimization problem and the total weighted completion time minimization problem remain polynomially solvable. Toksar and Guner [6] considered the parallel machine earliness/tardiness (ET) scheduling with simultaneous effects of learning and linear deterioration, sequence-dependent setups, and a common due-date for all jobs. They introduced a mixed nonlinear integer programming formulation for the problem. Wang and Cheng [7] considered the machine scheduling problems with the effects of deterioration and learning. They

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0307-904X/$ - see front matter 
doi:10.1016/j.apm.2010.09.028

introduced polynomial solutions for some single-machine problems and flow shop problems. The performance measures include makespan, total completion time, total weighted completion time, and maximum lateness. Wang et al. [8] considered the sing-machine problems of scheduling jobs with start-time increasing processing times (deterioration). The two objectives of the scheduling problems are to minimize the makespan and the total weighted completion time, respectively. Under the series-parallel graph precedence constraint assumption, they proved that the problems are polynomially solvable. Wang et al. [9] considered some single-machine scheduling problems with past-sequence-dependent setup times and the effects of deterioration and learning. They proved that the makespan minimization problem, the total completion time minimization problem, and the sum of the $\delta$th ($\delta \geqslant 0$) power of job completion times minimization problem can be optimally solved, respectively. They also proved that some special cases of the total weighted completion time minimization problem, the maximum lateness minimization problem and the number of tardy jobs minimization problem can be solved in polynomial time. Lee et al. [10] considered single-machine scheduling problem with a new deterioration model where the actual job processing time is a function of jobs already processed. They proved that the makespan minimization problem can be solved in polynomial time. Cheng et al. [11,12] and Wu and Lee [13] considered some scheduling problems with the actual job processing time is a function of jobs already processed. Wu and Lee [14] considered single-machine group scheduling problems with deteriorating setup times and job processing times. They proved that the makespan minimization problem remains polynomially solvable when the deterioration is present. They also showed that the sum of completion times problem is polynomially solvable when the numbers of jobs in each group are equal. Toksar and Guner [15] considered parallel machine earliness/tardiness (ET) scheduling problem with the effects of learning and deterioration. They presented a mixed nonlinear integer programming formulation for the simultaneous effects of learning and linear deterioration on parallel machine scheduling with sequence-dependent setups, and a common due-date for all jobs. Wang et al. [16] considered the single-machine group scheduling problems with deteriorating setup times and job processing times. They showed that the makespan minimization problem remains solvable in polynomial time when general linear deterioration and group technology are considered simultaneously. Wang et al. [17] considered the single-machine scheduling problem with a time-dependent deterioration effect. They showed that the makespan minimization problem remains polynomially solvable. They also showed that an optimal schedule of the total completion time minimization problem is V-shaped with respect to normal job processing times. Ji and Cheng [18] considered parallel-machine scheduling problems with simple linear deteriorating jobs. The objectives are to minimize the makespan, total machine load, and total completion time. They showed that all the problems are strongly NP-hard with an arbitrary number of machines and NP-hard in the ordinary sense with a fixed number of machines. Ng et al. [19] considered a two-machine flow shop scheduling problem to minimize the total completion time with proportional linear deterioration. They derived several dominance properties, some lower bounds, and an initial upper bound and applied them in a proposed branch-and-bound algorithm to search for the optimal solution. Yin et al. [20] considered the same model of Wang et al. [17]. They showed by an example that Theorem 3 and Corollary 2 in Wang et al. [17] are incorrect.

In this paper, we consider the single machine total completion minimization scheduling problem with a time-dependent deterioration proposed by Wang et al. [17]. The time-dependent deterioration can be described by the following example. There are some products that need to be processed by a cutting tool. Because of wear of the cutting tool, the time required for processing a single product increases with respect to the processing time of products already executed (Wang et al. [17]).

The paper is organized as follows: In Section 2, we formulate the model. In Section 3, we develop a mixed integer programming formulation for the problem. In Section 4, use the SPT rule as a heuristic for the general cases and analyze its worst-case error bound, we also propose two heuristic algorithms utilized the V-shaped property for the total completion time minimization problem, and followed by computational experiments. The last section is the conclusion.

## 2. Problem formulation

There are a single machine and a set $J = \{J_1, J_2, \ldots, J_n\}$ of $n$ independent jobs. All jobs are available for processing at time 0. The machine can handle one job at a time and preemption is not allowed. Associated with each job $J_j (j = 1, 2, \ldots, n)$ there is a normal processing time $p_j$ and $p_{[r]}$ be the normal processing time of a job if scheduled in the $r$th position in a sequence. Let $p_{j\,r}$ be the processing time of job $J_j$ if it is scheduled in position $r$ in a sequence. As in Wang et al. [17], we consider a time-dependent deterioration model:

$$p_{jr} = p_j\left(1 + p_{[1]} + p_{[2]} + \cdots + p_{[r-1]}\right)^a, \tag{1}$$

where $a \geqslant 0$ is the deterioration index, and $\sum_{i=1}^{0} p_{[i]} := 0$.

For a given schedule $\pi = [J_1, J_2, \ldots, J_n]$, $C_j = C_j(\pi)$ represents the completion time of job $J_j$. Let $\sum C_j$ represent the total completion time of a given permutation. In the remaining part of the paper, all the problems considered will be denoted using the three-field notation scheme $\alpha|\beta|\gamma$ introduced by Graham et al. [21].

## 3. A non-linear mathematical programming model

In this proposed model, there are $n2 + 6n$ variables and $8n$ constraints (for assigned to machine), where n denotes the number of jobs. The parameters and variables in the model are described below and then the proposed model is given.