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Recursive state estimation for hybrid systems

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ABSTRACT

The paper deals with recursive state estimation for hybrid systems. An unobservable state of such systems is changed both in a continuous and a discrete way. Fast and efficient online estimation of hybrid system state is desired in many application areas. The presented paper proposes to look at this problem via Bayesian filtering in the factorized (decomposed) form. General recursive solution is proposed as the probability density function, updated entry-wise. The paper summarizes general factorized filter specialized for (i) normal state-space models; (ii) multinomial state-space models with discrete observations; and (iii) hybrid systems. Illustrative experiments and comparison with one of the counterparts are provided.

1. Introduction

Systems whose state is changing dynamically continuously in time and also switching among several discrete values are understood as hybrid systems. A state of a hybrid system is modeled by continuous variables within several discrete modes, among them a system is switching. Usually system parameters are changing according to a particular mode. Hybrid systems are widely used in many fields of signal processing (target tracking, medicine, speech recognition, traffic control etc.). Fast and efficient online estimation of their state is desired in some of these areas.

A lot of works are devoted to state estimation of hybrid systems. One of the well-known approaches dealing with switching systems with Gaussian linear and discrete states is the interactive multiple model (IMM) algorithm [1]. It performs classical Kalman filter [2] for each mode under assumption that this particular mode is a right one at current time step. Then the IMM algorithm computes a weighted combination of updated state estimates produced by all the filters yielding a final Gaussian mean and covariance. This mixed state estimate is taken as the initial one for the next time step. The weights are chosen according to the probabilities of the models, which are computed in filtering step of the algorithm.

The paper [3] proposes the exact filter for a specialized hybrid system state. The reference probability method for hidden Markov models (HMM) is employed. The solution is presented as Gaussian sum with explicitly computed specific weights, means and variances. However, a number of statistics grows geometrically in time, and provided results are restricted only by 15 time steps. The approach [4] considers another special case of a dynamic linear state-space model with measurement matrices switching according to time-varying independent random process. The updating of probabilities is derived as an application of Bayes rule to the weighted observation model. The estimation of the normal state is shown as extension of the classical Kalman filter with involved weighted combinations of the gain-adjusted innovations.

Iterative techniques for jump Markov linear systems are nicely presented in [5]. The algorithms are derived to obtain the marginal maximum a posteriori sequence estimate of the finite state Markov chain. The paper [6] is concerned with optimal

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filtering for hybrid systems with non-Gaussian noises. The derived filter is optimal in the sense of the most probable trajectory (MPT) estimate. The state and the observation are considered as a pair of deterministic processes with switching coefficient as a random process. Despite the claimed generality of solution, this can restrict application in practice. The paper [7] proposes mixture Kalman filter based on a special sequential Monte Carlo method using a random mixture of Gaussian distributions for approximation of target posterior distribution. The approach deals with conditional dynamic linear models (CDLM) with mixed Gaussian noises defined via known indicator process. The weighted sample of the indicators is used within the proposed effective filter. A series of other research in the field of nonlinear hybrid systems [8] and online real-time state estimation [9] can be also found.

The presented paper is focused on modeling of system states as conditionally dependent entries of the state vector. Their entry-wise *recursive* estimation is subsequently reached via factorization of the state-space model and prior distributions for Bayesian filtering [10]. A part of the work dealing with estimation of discrete state is also closely related to algorithms based on hidden Markov models (HMM) theory [11]. However, these algorithms run in offline mode supported by Monte Carlo computations. Important features of the proposed theory are that:

- the algorithms used run in online mode,
- numerical procedures are applied only in that parts, which cannot be computed analytically. In this way the amount of computations as well as the risk of collapsing is minimized,
- general probabilistic approach is universal for the distributions used,
- it opens a way to recursive estimation of discrete system modes dependent on evolution of continuous states. This is planned for future research.

The paper is structured as follows. Necessary preliminaries are provided in Section 2. Section 3 presents general probabilistic solution of the factorized form of Bayesian filtering. The paper summarizes general factorized filter specialized for (i) normal state-space models in Section 4; (ii) multinomial state-space models with discrete-valued observations in Section 5 and (iii) hybrid systems in Section 6. Section 7 demonstrates examples with real data and comparison with the IMM filter. Remarks in Section 8 close the paper. Derivations of the proposed formulas are provided in Appendix A.

2. Preliminaries

2.1. State-space model

The system is described by the state-space model in the form of the following conditional probability (density) functions (p(d)fs) for simplicity denoted as pdfs within this paper

$$\textit{observation model} \quad f(y_t|x_t,u_t), \tag{1}$$
$$\textit{state evolution model} \quad f(x_{t+1}|x_t,u_t), \tag{2}$$

where the system output y_t and the control input u_t are measured at discrete time moments $t = \{1,\ldots,T\} \equiv t^*$. In general, the variables are column vectors such that $y_t = [y_{1;t},\ldots,y_{Y;t}]'$,$u_t = [u_{1;t},\ldots,u_{U;\,t}]'$. The system state $x_t = [x_{1;t},\ldots,x_{X;t}]'$ is not directly observed and has to be estimated in an online (*recursive*) mode.

2.2. Bayesian filtering

Bayesian filtering, estimating the system state, includes the following coupled formulas.

Data updating

$$f(x_t|D(t)) = \frac{f(y_t|x_t,u_t)f(x_t|D(t-1))}{\int f(y_t|x_t,u_t)f(x_t|D(t-1))dx_t} \propto f(y_t|x_t,u_t)f(x_t|D(t-1)), \tag{3}$$

incorporates information contained in observations $D(t) = (d_1,\ldots,d_t)$, where $d_t \equiv (y_t,u_t)$. Relation (3) also comprises the natural conditions of control [12], according to those

$$f(x_t|u_t,D(t-1)) = f(x_t|D(t-1)).$$

Time updating

$$f(x_{t+1}|D(t)) = \int f(x_{t+1}|x_t,u_t)f(x_t|D(t))dx_t, \tag{4}$$

fulfills state prediction. The prior pdf $f(x_1|D(0))$ which expresses the subjective prior knowledge on the system initial state starts the recursions. Application of (3,4) to linear Gaussian state-space model provides Kalman filter [12].