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Anti-plane elastodynamic analysis of cracked graded orthotropic layers with viscous damping

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ABSTRACT

Stress analysis is carried out in a graded orthotropic layer containing a screw dislocation undergoing time-harmonic deformation. Energy dissipation in the layer is modeled by viscous damping. The stress fields are Cauchy singular at the location of dislocation. The dislocation solution is utilized to derive integral equations for multiple interacting cracks with any location and orientation in the layer. These equations are solved numerically thereby obtaining the dislocation density function on the crack surfaces and stress intensity factors of cracks. The dependencies of stress intensity factors of cracks on the excitation frequency of applied traction and material properties of the layer are investigated. The analysis allows the determination of natural frequencies of a cracked layer. Furthermore, the interactions of two cracks having various configurations are studied.

1. Introduction

Functionally graded materials (FGMs) may be tailored to withstand high temperatures, harsh environments and severe stress levels, simultaneously. The widespread application of FGM for manufacturing mechanical components and also as thermal/wear resistant coating, for instance, in aerospace and automotive industries, makes thorough understanding of their behavior under various types of loads, such as periodic one of importance. In the area of fracture mechanics, a large number of articles deal with the stress analysis in media with FG constituents containing cracks. A brief review of investigations concerned with the dynamic anti-plane deformation of such structures is carried out here. Ma et al. [1], considered two collinear identical cracks in an isotropic FGM layer sandwiched between two isotropic dissimilar half-planes. The cracks were under constant anti-plane time-harmonic traction. A medium with the above composition, wherein defects were two interfacial coaxial cracks with different lengths, was analyzed by Ma et al. [2]. In another article Ma et al. [3] examined the problem of an orthotropic graded plane containing a crack in a direction of principal material orthotropy. In all the above cited studies, integral transforms and Schmidt's method were adopted to determine stress intensity factor at a crack tip. An isotropic half-plane reinforced by a FGM layer and weakened by an arbitrarily oriented crack under impact anti-plane traction was solved by Choi [4]. A meshless local boundary integral equation formulation for dynamic analysis of cracks in FGMs under anti-plane deformation was developed by Sladek et al. [5]. FGM coatings, due to the processing techniques used in their manufacturing, are usually orthotropic Dag et al. [6]. The static and dynamic anti-plane problems of an isotropic layer reinforced by an orthotropic FGM coating with periodic array of parallel cracks perpendicular to the interface was solved by Chen [7]. An orthotropic FGM layer with an edge or embedded crack perpendicular to the boundary under impact anti-plane traction was the subject of study by Chen and Liu [8]. In the above mentioned impact problems, Fourier and Laplace transforms were employed to obtain stress intensity factor at the crack tips.

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In the present article, the solution of a screw dislocation with time-harmonic excitation is derived in an orthotropic graded layer. In practice, energy dissipation occurs in the course of dynamic deformation of structures. Several investigators, in the analysis of structures composed of FGMs in the absence of defects, modeled this phenomenon by a viscous damping see e.g., Elishakoff et al. [9] and Kapuria et al. [10]. However, the structural damping was not taken into account in the previous studies of dynamic crack problems. To amend the shortcoming viscous damping is included to model energy dissipation under time-harmonic deformation of the layer. Two solutions in integral and series forms are obtained for stress components exhibiting Cauchy type singularity at the location of dislocation. The dislocation solution and Buckner's principle are employed to derive integral equations for dislocation density function in a graded strip weakened by multiple cracks. These equations are then solved numerically and the results are employed to determine stress intensity factor at the crack tips. The result of the study may be readily used to obtain Fourier series solutions for a cracked layer subjected to any distribution of self-equilibrating anti-plane periodic loads. Moreover, the solution encompasses those of cracked infinite- and half-plane regions under the aforementioned situation.

2. Orthotropic FGM strip with screw dislocation

A layer with thickness h made up of an orthotropic FGM, wherein material properties vary continuously in the thickness direction, is under consideration, Fig. 1. Moreover, x and y axes are taken as directions of principal material orthotropy. A Volterra-type screw dislocation with the line of dislocation coinciding with the y-axis is located at distance h_1 from the x-axis, Fig. 1. The displacement components in anti-plane deformation are

$$U = 0, \quad V = 0, \quad W = W(x, y, t). \tag{1}$$

Utilizing strain-deformation relationships in linear elasticity, the non-zero strain components become

$$\gamma_{zx} = \frac{\partial W}{\partial x}, \quad \gamma_{zy} = \frac{\partial W}{\partial y}. \tag{2}$$

Substituting Eq. (2) into Hooke's law for the orthotropic FGM layer, leads to the stress components in terms of displacement field

$$\hat{\sigma}_{zx} = \mu_x(y)\frac{\partial W}{\partial x}, \quad \hat{\sigma}_{zy} = \mu_y(y)\frac{\partial W}{\partial y}, \tag{3}$$

where $\mu_x(y)$ and $\mu_y(y)$ are the shear moduli of FGM in the x- and y-directions, respectively. The equation for anti-plane motion of a body with internal viscous dissipation reads

$$\frac{\partial \hat{\sigma}_{zx}}{\partial x} + \frac{\partial \hat{\sigma}_{zy}}{\partial y} = \rho(y)\frac{\partial^2 W}{\partial t^2} + \eta(y)\frac{\partial W}{\partial t}, \tag{4}$$

where $\rho(y)$ and $\eta(y)$ are the mass density and the viscous damping coefficient per unit volume of material, respectively. At the outset of dynamic loading transient effects prevail. The last term in Eq. (4), however, gradually eliminates these effects making time-harmonic motion physically plausible. Eq. (4) in view of Eq. (3) becomes

$$\frac{\mu_x(y)}{\mu_y(y)}\frac{\partial^2 W}{\partial x^2} + \frac{\partial^2 W}{\partial y^2} + \frac{\mu'_y(y)}{\mu_y(y)}\frac{\partial W}{\partial y} = \frac{\rho(y)}{\mu_y(y)}\frac{\partial^2 W}{\partial t^2} + \frac{\eta(y)}{\mu_y(y)}\frac{\partial W}{\partial t}. \tag{5}$$

For the layer depicted in Fig. 1, the boundary, continuity and limiting conditions may be expressed as

$$\begin{aligned} &\widehat{\sigma}_{yz}(x, 0, t) = 0, \quad \widehat{\sigma}_{yz}(x, h, t) = 0, \\ &W(0^+, y, t) - W(0^-, y, t) = B_z(t)H(y - h_1), \\ &\widehat{\sigma}_{xz}(0^+, y, t) = \widehat{\sigma}_{xz}(0^-, y, t), \\ &\lim_{|x|\to\infty} W = 0, \end{aligned} \tag{6}$$

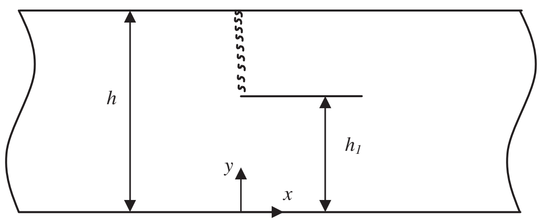

Fig. 1. Schematic view of the FGM layer with screw dislocation.