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Chaos synchronization between two different chaotic systems with uncertainties, external disturbances, unknown parameters and input nonlinearities

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ABSTRACT

In this paper, the problem of chaos synchronization between two different uncertain chaotic systems with input nonlinearities is investigated. Both master and slave systems are perturbed by model uncertainties, external disturbances and unknown parameters. The bounds of the model uncertainties and external disturbances are assumed to be unknown in advance. First, a simple linear sliding surface is selected. Then, appropriate adaptive laws are derived to tackle the model uncertainties, external disturbances and unknown parameters. Subsequently, based on the adaptive laws and Lyapunov stability theory, a robust adaptive sliding mode control law is designed to guarantee the existence of the sliding motion. Two illustrative examples are presented to verify the usefulness and applicability of the proposed technique.

1. Introduction

Chaotic systems are very complex dynamical systems that possess some special attributes such as extremely sensitivity to initial conditions, broad Fourier transform spectra, fractal properties of the motion in phase space and strange attractors. Due to its useful applications in biological systems, chemical reactions, information processing, secure communication, economical systems and power convertors, synchronization of chaotic systems has become an interesting and important topic among researchers of mathematics, physics and engineering sciences in recent years and a wide variety of control approaches, such as fuzzy sliding mode control [1,2], adaptive control [3], optimal control [4], nonlinear feedback control [5] and H_∞ control [6], have been proposed to synchronize chaotic systems.

However, all of these works and many others in the literature have focused on the study of chaos synchronization between two chaotic systems without model uncertainties and external disturbances. While, in real world applications, due to un-modeled dynamics, system structural variations and measurement and environment noises the uncertainties and external disturbances affect chaotic systems. In this regard, a number of researchers have paid their attention to the synchronization of uncertain chaotic systems [7–14]. Although the works [7–14] have solved the problem of synchronization of uncertain chaotic systems, but the chaotic systems considered in these studies have fully (or partially) known parameters. Nevertheless, in practical situations, it is hard to exactly determine the values of the parameters of the chaotic systems in advance. To solve this problem, some scholars have introduced several techniques for synchronization of chaotic systems with unknown parameters, including sliding mode control [15–19], adaptive control [20–23], optimal control [24–26],

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backstepping design [27] and fuzzy control [28]. However, these studies have not considered the effects of both model uncertainties and external disturbances in the dynamics of the chaotic systems.

On the other hand, when a controller is implemented in practice, limitations of actuators cause some nonlinearities, such as backslash, hysteresis, saturation and dead-zone, in the control input. Since chaotic systems are very sensitive to the variations of any system parameters, the presence of the nonlinearities in the control input can lead to unpredictable behaviors in the chaotic systems and even break the synchronization. Therefore, the effect of the input nonlinearities should not be neglected in designing and realizing the controller in applications. Li and Chang [29] have developed an adaptive sliding mode controller for synchronizing a class of identical chaotic systems with uncertainties and nonlinear control inputs. The proposed method is not applicable for synchronization of two different chaotic systems. Lin et al. [30] have proposed adaptive sliding mode observers to synchronize a class of identical chaotic systems with input nonlinearity. In [31], the sliding mode control theory has been used to design a controller for synchronization of two deterministic different chaotic systems with input nonlinearities. Kebriaei and Yazdanpanah [32] have derived an adaptive sliding mode controller to synchronize an special class of different uncertain chaotic systems with fully known parameters and input nonlinearities.

However, to the authors' best knowledge, the problem of synchronization of two different chaotic systems with model uncertainties, external disturbances, unknown parameters and input nonlinearities is remaining as an open and challenging problem yet. Therefore, we aim to solve this problem in the present paper. We consider two n-dimensional different chaotic systems with model uncertainties, external disturbances and unknown parameters in both master and slave systems. Besides, we assume that the control input, attached to the slave system, contains nonlinearities. The bounds of the uncertainties and external disturbances are supposed to be unknown in priori. Appropriate adaptive laws are designed to tackle the model uncertainties, external disturbances and unknown parameters. Based on the adaptive laws and Lyapunov stability theory, a robust adaptive sliding mode controller (RASMC) is designed and its robustness and stability are analytically proved. Finally, we present some numerical simulations to demonstrate the feasibility and usefulness of the proposed RASMC.

The rest of this paper is organized as follows. In Section 2, the structures of the master and slave systems are given and the synchronization problem is formulated. Section 3 deals with the design procedure of the proposed RASMC. Some numerical simulations are included in Section 4. Finally, conclusions are presented in Section 5.

2. System modeling and problem formulation

In this paper, the following class of n-dimensional master and slave chaotic systems with model uncertainties, external disturbances and unknown parameters are taken into account.

Master system:

$$\begin{aligned}
\dot{x}_1(t) &= f_1(x_1,x_2,\dots,x_n) + F_1(x_1,x_2,\dots,x_n)\theta + \Delta f_1(x_1,x_2,\dots,x_n,t) + d_1^m(t),\\
\dot{x}_2(t) &= f_2(x_1,x_2,\dots,x_n) + F_2(x_1,x_2,\dots,x_n)\theta + \Delta f_2(x_1,x_2,\dots,x_n,t) + d_2^m(t),\\
&\vdots\\
\dot{x}_n(t) &= f_n(x_1,x_2,\dots,x_n) + F_n(x_1,x_2,\dots,x_n)\theta + \Delta f_n(x_1,x_2,\dots,x_n,t) + d_n^m(t).
\end{aligned} \tag{1}$$

Slave system:

$$\begin{aligned}
\dot{y}_1(t) &= g_1(y_1,y_2,\dots,y_n) + G_1(y_1,y_2,\dots,y_n)\psi + \Delta g_1(y_1,y_2,\dots,y_n,t) + d_1^s(t) + \phi_1(u_1),\\
\dot{y}_2(t) &= g_2(y_1,y_2,\dots,y_n) + G_2(y_1,y_2,\dots,y_n)\psi + \Delta g_2(y_1,y_2,\dots,y_n,t) + d_2^s(t) + \phi_2(u_2),\\
&\vdots\\
\dot{y}_n(t) &= g_n(y_1,y_2,\dots,y_n) + G_n(y_1,y_2,\dots,y_n)\psi + \Delta g_n(y_1,y_2,\dots,y_n,t) + d_n^s(t) + \phi_n(u_n),
\end{aligned} \tag{2}$$

where $x(t) = [x_1,x_2,\dots,x_n]^T \in R^{n\times 1}$ is the state vector of the master system, $f_i(x) : R^{n\times 1} \to R$, $i = 1, 2, \dots, n$ is a continuous nonlinear function, $F_i(x) : R^{n\times 1} \to R^{1\times m}$, $i = 1, 2, \dots, n$ is the ith row of an $n \times m$ matrix whose elements are continuous nonlinear functions, $\theta \in R^{m\times 1}$ is an $m \times 1$ unknown parameter vector of the master system, $\Delta f(x,t) = [\Delta f_1(x,t), \Delta f_2(x,t),\dots,\Delta f_n(x,t)]^T : R^{n\times 1} \times R^+ \to R^{n\times 1}$ and $d^m(t) = \left[d_1^m(t), d_2^m(t),\dots,d_n^m(t)\right]^T : R^+ \to R^{n\times 1}$ are the vectors of unknown model uncertainties and external disturbances of the master system, respectively, $y(t) = [y_1,y_2,\dots,y_n]^T \in R^{n\times 1}$ is the state vector of the slave system, $g_i(y) : R^{n\times 1} \to R$, $i = 1, 2, \dots, n$ is a continuous nonlinear function, $G_i(y) : R^{n\times 1} \to R^{1\times m}$, $i = 1, 2, \dots, n$ is the ith row of an $n \times m$ matrix whose elements are continuous nonlinear functions, $\psi \in R^{m\times 1}$ is an $m \times 1$ unknown parameter vector of the slave system, $\Delta g(y,t) = [\Delta g_1(y,t), \Delta g_2(y,t),\dots,\Delta g_n(y,t)]^T : R^{n\times 1} \times R^+ \to R^{n\times 1}$ and $d^s(t) = \left[d_1^s(t), d_2^s(t),\dots,d_n^s(t)\right]^T : R^+ \to R^{n\times 1}$ are the vectors of unknown model uncertainties and external disturbances of the slave system, respectively, $u(t) = [u_1,u_2,\dots,u_n]^T \in R^{n\times 1}$ is the vector of control inputs and $\phi_1(u_1), \phi_2(u_2), \dots, \phi_n(u_n)$ with $\phi_i(0) = 0$, $i = 1, 2, \dots, n$ are continuous nonlinear functions inside the sector $[\rho_i, \mu_i]$, $i = 1, 2, \dots, n$, $\rho_i > 0$, i.e.

$$\rho_i u_i^2 \leqslant u_i\phi_i(u_i) \leqslant \mu_i u_i^2, \quad i = 1,2,\dots,n. \tag{3}$$