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Some new results in linear programs with trapezoidal fuzzy numbers: Finite convergence of the Ganesan and Veeramani's method and a fuzzy revised simplex method

A. Ebrahimnejad

Islamic Azad University, Qaemshahr Branch, Department of Mathematics, Qaemshahr, Iran

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ABSTRACT

In a recent paper, Ganesan and Veermani [K. Ganesan, P. Veeramani, Fuzzy linear programs with trapezoidal fuzzy numbers, Ann. Oper. Res. 143 (2006) 305–315] considered a kind of linear programming involving symmetric trapezoidal fuzzy numbers without converting them to the crisp linear programming problems and then proved fuzzy analogues of some important theorems of linear programming that lead to a new method for solving fuzzy linear programming (FLP) problems. In this paper, we obtain some another new results for FLP problems. In fact, we show that if an FLP problem has a fuzzy feasible solution, it also has a fuzzy basic feasible solution and if an FLP problem has an optimal fuzzy solution, it has an optimal fuzzy basic solution too. We also prove that in the absence of degeneracy, the method proposed by Ganesan and Veermani stops in a finite number of iterations. Then, we propose a revised kind of their method that is more efficient and robust in practice. Finally, we give a new method to obtain an initial fuzzy basic feasible solution for solving FLP problems.

1. Introduction

Fuzzy linear programming (FLP) has been developed for treating uncertainty in the setting of optimization problems. In recent years, various attempts have been made to study the solution of FLP problems, either from theoretical or computational point of view. The concept of fuzzy linear programming on general level was first proposed by Tanaka et al. [1] in the framework of the fuzzy decision of Bellman and Zadeh [2]. Afterwards many authors have considered various kinds of the FLP problems and have proposed several approaches for solving these problems [3–7]. Some authors have used the concept of comparison of fuzzy numbers and linear ranking function to solve the fuzzy linear programming problems. Of course, ranking functions have been proposed by researchers to suit their requirements of the problem under consideration and conceivably there are no generally accepted criteria for application of ranking functions. Nevertheless, usually in such methods authors define a crisp model which is equivalent to the FLP problem and then use optimal solution of the model as the optimal solution of the FLP problem. Maleki et al. [3] using the concept of comparison of fuzzy numbers, proposed a new method for solving fuzzy number linear programming (FNLP) problems. Then Mahdavi-Amiri and Nasseri [4] used the certain linear ranking function to define the dual of FNLP problems as FNLP problems again that lead to an efficient method called the dual simplex algorithm [8] for solving FNLP problems. Also, Mahdavi-Amiri and Nasseri [5] proposed another approach to define dual of FNLP problems as fuzzy variable linear programming (FVLP) problems leading to a dual simplex algorithm for solving FVLP problems. Then, Ebrahimnejad et al. [9] introduced another efficient method namely primal–dual simplex algorithm to

E-mail addresses: a.ebrahimnejad@srbiau.ac.ir, aemarzoun@gmail.com

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obtain a fuzzy solution of FVLP problems. Also, Ebrahimnejad and Nasseri [10] used the complementary slackness to solve FNLP and FVLP problems without the need of a simplex tableau. Then Nasseri and Ebrahimnejad [11] proposed a fuzzy primal simplex algorithm for solving the flexible linear programming problem and then suggested the fuzzy primal simplex method to solve the flexible linear programming problems directly without solving any auxiliary problem. Hosseinzadeh Lofi et al. [12] discussed full fuzzy linear programming (FFLP) problems of which all parameters and variable are triangular fuzzy numbers. They used the concept of the symmetric triangular fuzzy number and introduced an approach to defuzzify a general fuzzy quantity. For such a problem, first, the fuzzy triangular number is approximated to its nearest symmetric triangular number, with the assumption that all decision variables are symmetric triangular. Kumar et al. [13] proposed a new method to find the fuzzy optimal solution of same type of fuzzy linear programming problems. Ebrahimnejad [14] based on fuzzy simplex algorithms for solving fuzzy number linear programming and using the general linear ranking functions on fuzzy numbers generalized a concept of sensitivity analysis in FNLP problems.

Recently, Ganesan and Veeramani [15] introduced a new type of fuzzy arithmetic for symmetric trapezoidal fuzzy numbers and proposed a method for solving FLP problems without converting them to the crisp linear programming problems. Ebrahimnejad et al. [16] generalized their method for solving bounded linear programming problems with symmetric trapezoidal fuzzy numbers. Nasseri and Mahdavi-Amiri [17] and Nasseri et al. [18] used their results to define the dual of fuzzy linear programming. In this paper based on this new arithmetic, we obtain some other new results for FLP problems. We also prove that in the absence of degeneracy, the method proposed by Gaesan and Veermani [15] stops in a finite number of iterations. Then we propose a revised kind of their method that is more efficient and robust in practice.

This paper is organized as follows: in Section 2, we give some necessary concepts and backgrounds of fuzzy arithmetic. In Section 3, we first review the method proposed by Gaesan and Veermani [15] for solving FLP problems and then prove some new results about these problems and give a tableau format of the fuzzy primal method. The fuzzy revised simplex algorithm to solve FLP problems is given in Section 4. We propose a method for solving FLP problems with the assumption that an initial basic feasible solution is not readily available in Section 5. Finally, we conclude in Section 6.

2. Preliminaries

In this section we introduce some of the basic terminologies of fuzzy set theory and the main concepts needed in the rest of the paper.

Definition 2.1. Let $\mathbb{R}$ be the universal set. $\tilde{a}$ is called a fuzzy set in $\mathbb{R}$ if $\tilde{a}$ is a set of ordered pairs $\tilde{a} = \{(x, \mu_{\tilde{a}}(x)) | x \in \mathbb{R}\}$, where $\mu_{\tilde{a}}(.)$ is membership function of $\tilde{a}$ and assigns to each element $x \in \mathbb{R}$, a real number $\mu_{\tilde{a}}(x)$ in the interval [0,1].

Definition 2.2. The α-cut or α-level of a fuzzy set $\tilde{a}$ is defined as an ordinary set $[\tilde{a}]_\alpha$ for which the degree of its membership function exceeds the level α, that is, $[\tilde{a}]_\alpha = \{x \in \mathbb{R} | \mu_{\tilde{a}}(x) \geqslant \alpha\}$.

Definition 2.3. The support of a fuzzy set $\tilde{a}$ is a set of elements in $\mathbb{R}$ for which $\mu_{\tilde{a}}(x)$ is positive, that is, $supp\,\tilde{a} = \{x \in \mathbb{R} | \mu_{\tilde{a}}(x) > 0\}$.

Definition 2.4. A fuzzy set $\tilde{a}$ is called convex if for each $x, y \in \mathbb{R}$ and each $\lambda \in [0,1]$, $\mu_{\tilde{a}}(\lambda x + (1-\lambda)y) \geqslant \min\{\mu_{\tilde{a}}(x), \mu_{\tilde{a}}(y)\}$.

Definition 2.5. A fuzzy set $\tilde{a}$ is called a normal fuzzy set if $sup\{\mu_{\tilde{a}}(x) | x \in \mathbb{R}\} = 1$.

Definition 2.6. A fuzzy number is a convex normalized fuzzy set of the real line $\mathbb{R}$; whose membership function is piecewise continuous.

Definition 2.7. An LR type flat fuzzy number [19], is denoted as $\tilde{a} = (a^L, a^U, \alpha, \beta)_{LR}$, if

$$\mu_{\tilde{a}}(x) = \begin{cases} L\left(\frac{a^L - x}{\alpha}\right) & \text{for} \quad a^L - \alpha \leqslant x \leqslant a^L, \\ 1 & \text{for} \quad a^L \leqslant x \leqslant a^U, \\ R\left(\frac{x - a^U}{\beta}\right) & \text{for} \quad a^U \leqslant x \leqslant a^U + \beta, \\ 0 & \text{else}, \end{cases} \tag{1}$$

where the symmetric non-increasing function $L : [0, \infty) \to [0, 1]$ is the left shape function, that L (0) = 1. Also, a right shape function R(.) is similarly defined as L(.).