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Exact solution for Transient bending of a circular plate integrated with piezoelectric layers

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ABSTRACT

This paper presents the exact, explicit solution for the transient motion of a circular plate surface bonded by two piezoelectric layers, based on Kirchhoff plate model. The distribution of eclectic potential along the thickness direction is simulated by a quadratic function so that the Maxwell static electricity equation is satisfied. The piezoelectric layers are electrically grounded over the edge and electrodes at the two surfaces of the piezoelectric layers are shortly connected. The differential equations of motion are solved for simply supported and clamped boundary conditions. The solutions are expressed by elementary Bessel functions and obtained via exact inverse Laplace transform.

1. Introduction

In the recent years the analysis of a coupled piezoelectric structure has been extensively researched because piezoelectric materials are more widely used either as actuators or sensors. Examples consist of the analytical modeling of a structures with surface-bonded or embedded piezoelectric sensors and actuators [1,2] and use of piezoelectric materials in composite laminates and for vibration control [3,4]. Wang and Quek [5] presented their study on the free vibration of a piezoelectric sandwich beam structure, in which the piezoelectric effect on the resonance frequencies of the structure and the distribution of the electric potential are investigated. Also, they analyzed free vibration of piezoelectric coupled circular plate [6].

For isotropic elastic materials, a good account of axisymmetric static bending of circular plates is recorded in Ref. [7]. Similar problems for piezoelectric structures are analyzed in Refs. [8–19]. Zhang et al investigated the static and transient bending of a piezoelectric circular plate under axisymmetric, mechanical loading, electrically grounded over the whole surface and built-in or simply supported at the edge [10]. Wu et al presented a vibration analysis of a circular steel substrate surface bonded by a piezoelectric layer with open circuit [11]. Haojiang et al. used finite Hankel transforms and derived the axisymmetric state space formulation of piezoelectric laminated circular plates based on a three dimensional elastic theory [12]. Alibeigloo and Simintan using differential quadrature investigated the behavior of functionally graded circular and annular plates integrated with sensor and actuator layers [13]. Vibration analysis of piezoelectric coupled to the thick annular FGM plates subjected to different combinations of simply supported and clamped boundary conditions at the inner and outer edges of the annular plate on the basis of the Reddy's third-order shear deformation theory was presented by Hosseini-Hashemi et al. [14]. Huang and Shen [15] investigated the dynamics of an FG plate coupled with two monolithic piezoelectric layers at its top and bottom surfaces undergoing nonlinear vibrations in thermal environments. Based on Mindlin's plate theory, free vibration analysis of moderately thick shear deformable circular and annular functionally graded plates coupled

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with piezoelectric layers are presented by Ebrahimi et al. [16,17]. Lim and He [18] studied triple-layer actuators composed of a metallic plate with two transversely isotropic piezoelectric layers mounted on it. They showed that the electromechanical field can be recast into a transfer matrix form and further expressed as a closed-form solution in terms of the displacement components on the mid-plane. Sekouri et al. [19] developed an analytical approach for modeling a circular plate structure with integrated distributed piezoelectric actuators under static as well as dynamic mechanical or electrical loadings. They employed modal analysis for determining the natural frequencies and mode shapes of the structures while harmonic analysis was performed to analyze the steady-state behavior of the structures subjected to cyclic sinusoidal loads.

In this paper, the transient bending of a thin, circular plate coupled with piezoelectric layers under axisymmetric mechanical loading is presented for different boundary conditions. The piezoelectric layers are electrically grounded over the edge and electrodes at the two surfaces of the piezoelectric layers are shortly connected. The formulations are based on the classical plate theory (CPT). A consistent formulation that satisfies the Maxwell static electricity equation is presented for piezoelectric layers. The distribution of electric potential field along the thickness direction of the piezoelectric layers is simulated by a quadratic function and the solutions are expressed in terms of a single, elementary Bessel function and the explicit time history of the solution is obtained by precise inverse Laplace transformation. Thus, accurate results of system characteristics can easily be obtained applying simple numerical procedures.

2. Displacement and electric potential field for triple-layer circular plate

Geometry of the circular plate with a piezoelectric layer mounted on its surface is shown in Fig. 1. In the present study according to the ratio of the plate radius to its thickness, the Kirchhoff assumption for thin plates is applicable, whereby the shear deformation and rotary inertia can be omitted. Furthermore, the axisymmetric applied external load assumed.

Due to symmetry, the displacement and strains in the cylindrical coordinate system are expressed as:

$$u_z = u_z(r,t) = w(r,t), \tag{1}$$

$$u_r = u_r(r,t) = -z\frac{\partial u_z}{\partial r}, \tag{2}$$

$$u_\theta = 0, \tag{3}$$

where u_z, u_r and u_θ are the displacements in the transverse z-direction, radial r-direction, and tangential θ-direction of the plate, respectively. The poling direction of the piezoelectric material is assumed to be in the z-direction. The strain ε in the main plate and the piezoelectric layer with respect to the radial and tangential directions and the shear component are given by

$$\varepsilon_r = \frac{\partial u_r}{\partial r} = -z\frac{\partial^2 w}{\partial r^2}, \tag{4}$$

$$\varepsilon_\theta = \frac{u_r}{r} = -z\frac{\partial w}{r\partial r}, \tag{5}$$

$$\varepsilon_{r\theta} = 0. \tag{6}$$

Due to symmetry, the stress components in the main plate are expressed as [7]

$$\sigma_r^1 = \frac{E(\varepsilon_r + \upsilon\varepsilon_\theta)}{1-\upsilon^2}, \tag{7}$$

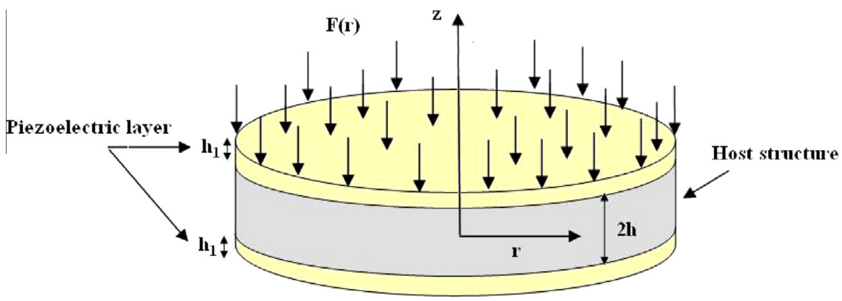

Fig. 1. The circular plate with two piezoelectric layers mounted on its surfaces.