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## Coupled heat and moisture transport in paper with application to a warm print surface

Paula Andrea Marin Zapata <sup>a,\*</sup>, Maurice Fransen <sup>b,1</sup>, Jan ten Thije Boonkkamp <sup>c</sup>, Louis Saes <sup>d</sup><sup>a</sup> Department of Mathematics and Computer Science, Eindhoven University of Technology, 5600 MB Eindhoven, The Netherlands<sup>b</sup> Mesoscopic Transport Phenomena, Department of Applied Physics, Eindhoven University of Technology, 5600 MB Eindhoven, The Netherlands<sup>c</sup> Centre for Analysis, Scientific computing and Applications (CASA), Department of Mathematics and Computer Science, Eindhoven University of Technology, 5600 MB Eindhoven, The Netherlands<sup>d</sup> Océ Technologies B.V., 5914 CA Venlo, The Netherlands

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### ABSTRACT

In this work we present a mathematical model describing the coupled heat and moisture transport in paper. The model is solved numerically and the numerical solution is used to study the interdependency of the moisture and temperature distribution in paper. The results show that variation with temperature of the saturated water vapor concentration and the sorption isotherm parameters are both important for inducing moisture desorption. It is also found that for steep relative humidity ramps moisture sorption generates temperature increments that slow down the sorption process itself. The model is also used to study the moisture gradients in a paper sheet inside a printer from Océ Technologies, which contains a warm print surface. The results predict changes in moisture content of only 0.2%, which suggests that no deformations are induced on the printed sheet.

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## 1. Introduction

The dimensional stability of paper, i.e., the property to maintain its original dimensions while being used, is of importance for the printing and copying industry. During printing, heating and adding ink or toner can change the moisture content of the paper sheets, thereby creating moisture gradients. These gradients are undesirable since they induce local expansion or compression of the fibers, generating mechanical instabilities like curls and cockles [1,2]. Such instabilities affect the quality of the prints, and since they can cause paper jams, they also affect the productivity of the printers.

To understand the moisture-induced deformation of paper, the heat and moisture transport processes need to be understood first. Several authors have derived moisture transport models for the final use of paper. Dano and Bourque [3] modeled the moisture diffusion through the thickness of a paper sheet, using their model to predict the formation of curls. In their work paper was modeled as a homogeneous medium, but as noted in [4] better results are obtained by modeling paper as a porous medium. More elaborate models considering paper as a composite of pores and fibers were presented in [5–8]. Although these models showed good agreement with experiments, they were restricted to isothermal conditions, which do not always apply to the printing and copying practice. Few non-isothermal models for moisture transport at the end use of paper have been developed. Bandyopadhyay and Ramarao [9] modeled the evolution of the temperature,

\* Corresponding author. Address: Bioquant, Heidelberg University, Im Neuenheimer Feld 267, 69120 Heidelberg, Germany.

E-mail addresses: [paula.marin@bioquant.uni-heidelberg.de](mailto:paula.marin@bioquant.uni-heidelberg.de) (P.A. Marin Zapata), [m.a.l.j.fransen@tue.nl](mailto:m.a.l.j.fransen@tue.nl) (M. Fransen), [tenthije@win.tue.nl](mailto:tenthije@win.tue.nl) (J. ten Thije Boonkkamp), [louis.saes@oce.com](mailto:louis.saes@oce.com) (L. Saes).

<sup>1</sup> Engineering Thermodynamics for Energy Systems, Department of Mechanical Engineering, Eindhoven University of Technology, 5600 MB Eindhoven, The Netherlands.

moisture and pressure in a paper sheet traveling through a hot roll nip. Since they worked with temperatures of about 200 °C, pores and fibers were assumed to be in thermodynamic equilibrium, which does not hold true for normal room temperatures. Foss et al., [10] modeled the heat and moisture transport in a paper sheet in response to variations of the relative humidity in the surroundings of the sheet. They studied temperature changes induced by moisture sorption, but the impact of temperature changes on the evolution of the moisture content was not clearly established. In order to study the moisture transport at the final use of printing and copying paper, a model that accounts for the interdependent relation between temperature and moisture under non-equilibrium conditions still needs to be developed.

In this work we derive a model for the coupled heat and moisture transport in paper. This model includes the effect of temperature changes on the transient moisture content of paper, as well as the effect of moisture sorption on the evolution of the temperature. The numerical solution of the model is used to study the relation between moisture and temperature, and to simulate an application in which a paper sheet is transported over a warm print surface. The remainder of this paper is organized as follows: Section 2 gives a detailed derivation of the governing equations for heat and moisture transport in paper. Section 3 subsequently describes the experimental procedures used to calculate some of the physical parameters, and Section 4 presents the numerical solution of the model. Based on the equations derived in Section 2, Section 5 presents a two-dimensional model for the application to a printer from Océ technologies. All simulation results are presented and discussed in Section 6, and finally, Section 7 presents a summary and conclusions.

## 2. Governing equations for heat and moisture transport in paper

Paper consists of cellulose fibers and often fillers and additives that are embedded in a complex solid matrix. Since this solid matrix is permeated by a network of interconnected pores, paper is considered a porous medium. Water molecules can be present in the pore space in the form of vapor or as free liquid. Additionally, since paper is a hygroscopic material, the solid matrix can contain water molecules bound by physicochemical forces to the surface of the fibers (freezing or non-freezing bound water [11]). Several moisture transport mechanisms can be identified inside hygroscopic materials [12,13]: diffusion of water vapor in the pores, surface diffusion (water adsorbed on the solid matrix), and capillary transport (free liquid water in the pores). Which forms of water are present and which transport mechanisms are significant depend greatly on the value of the relative humidity (RH).

In this work we model the transport processes at the final use of printing and copying paper. Since the operation range of printers is usually restricted to relative humidities below 75 to 80%, we restrict our model to  $RH < 75\%$ . For this range of relative humidity no free liquid water is expected to be present. Free liquid water can be generated inside the pore space by means of capillary condensation, a phenomenon that makes unsaturated vapor condensate as bulk liquid in channels of sufficiently small dimensions. Parker et al., [14] studied the effect of capillary condensation in paper and paperboard, concluding that this phenomena was negligible for RH below 80%. Based on this, paper can be modeled as a porous medium consisting of two phases: a fiber network and a pore space filled with air. In this section we derive the balance equations for each of these two phases using the framework derived in [15] to model transport in porous media with the method of Representative Elementary Volumes (REV).

### 2.1. Mass balances

If we denote the pore space by the subscript p and the solid matrix by the subscript f (fibers), neglecting possible convection, and assuming the volumetric fraction of pores and fibers to be constant in time and space, the balances of water in paper are given by

$$\eta \frac{\partial C_p}{\partial t} = \eta \nabla \cdot (\mathbf{D}_p^{\text{eff}} \nabla C_p) + \eta \Gamma_p - S_{pf}, \quad (1)$$

$$(1 - \eta) \frac{\partial C_f}{\partial t} = (1 - \eta) \nabla \cdot (\mathbf{D}_f^{\text{eff}} \nabla C_f) + (1 - \eta) \Gamma_f + S_{pf}, \quad (2)$$

where  $C_p$  [ $\text{kg m}^{-3}$ ] is the concentration of water vapor in the pores,  $C_f$  [ $\text{kg m}^{-3}$ ] is the concentration water bound to the fibers,  $0 < \eta < 1$  [-] is the volumetric fraction of pore space, also known as the porosity of the paper sheet,  $(1 - \eta)$  [-] is the volumetric fraction of fibers, the diagonal tensor  $\mathbf{D}_p^{\text{eff}}$  [ $\text{m}^2 \text{s}^{-1}$ ] is the effective diffusivity of water vapor in the pores, the diagonal tensor  $\mathbf{D}_f^{\text{eff}}$  is the effective diffusivity of liquid water bound to the fibers,  $\Gamma_p$  is the rate of production of water in the pores per unit volume of pores,  $\Gamma_f$  is the rate of production of water in the fibers per unit volume of fibers, and  $S_{pf}$  [ $\text{kg m}^{-3} \text{s}^{-1}$ ] is the rate of mass exchange from pores to fibers per unit volume of porous medium, in this case by sorption or desorption.

Neglecting possible addition of ink leads to  $\Gamma_p = \Gamma_f = 0$ . Since we restrict our model to relative humidities below 75%, we also neglect the diffusion of water bound to the fibers ( $\mathbf{D}_f^{\text{eff}} = 0$ ). The diffusivity of water molecules in paper fibers has been estimated in [9,10]. Bandyopadhyay et al., [9] reported a diffusivity of the order of  $10^{-14}$  [ $\text{m}^2 \text{s}^{-1}$ ] for relative humidities below 75%. They concluded that for the diffusivity in the fibers to be significant, the moisture content should be greater than 13%. In our case, such moisture contents would only be achieved for relative humidities above 83%. Similarly, by comparing their model against experimental results, Foss et al., [10] found a diffusivity of water in the fibers of  $3.8 \times 10^{-14}$  [ $\text{m}^2 \text{s}^{-1}$ ] for

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