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# Turbulent flow around single concentric long capsule in a pipe

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ABSTRACT

A numerical solution was developed for the equations governing the turbulent flow around single concentric long capsule in a pipe. First, a turbulence model was established for the concentric annulus between the capsule and the pipe to simulate the flow as axi-symmetric, two dimensional, steady flow without edge effect. Second, the same case was considered taking into account the edge effect. Finally, turbulence modelling was established to simulate the case as a three dimensional steady flow, with a view of investigating the validity of axi-symmetric flow assumption. Three different turbulence models were used: an algebraic model (Baldwin–Lomax model) and two types of two-equation models ($k$–$\varepsilon$ and $k$–$\omega$). Obtained results of pressure gradient along the capsule were compared with available experimental data to verify the used models. In addition, experimental data of the velocity profiles of other investigators were also used in this concern. The results predicted by the three different turbulence models were shown to agree well with the experimental data, though precision differed from one to another.

## 1. Introduction

Hydraulic capsule pipeline (HCP) is a transportation method, which uses floated capsules to transport cargoes through a pipeline. The water is used both to float (suspend) and push (drive) the capsules through the pipeline. Capsules travel at 1.8–3 m/s in the pipe. HCP is suitable for transporting bulk materials such as grains and other agricultural products, and municipal solid waste, which do not require high speed for delivery. HCP was first tested and studied in Canada at Alberta Research Center during the period 1958–1978 [1]. In 1991, the National Science Foundation in the United States established a Capsule Pipeline Research Center (CPRC) at the University of Missouri-Columbia (UMC) to develop various capsule pipeline technologies, including Hydraulic Capsule Pipeline (HCP), Pneumatic capsule Pipeline (PCP) and Coal Log Pipeline (CLP). It resulted in extensive research and development in HCP, bringing the technology close to commercial use [2,3]. Liu and Rhee [4] studied, theoretically and experimentally, the behavior of non-uniform-density capsules. Lenau and El-Bayya [5] presented a theoretical model for unsteady hydraulic capsule flow using the method of characteristics. Cheng and Liu [6] studied the tilt of stationary capsule in pipe and presented theoretical and experimental models for such capsule. Huang et al. [7] and Azpuz and Shirazi [8]investigated the effect of the drag reduction additives on the capsule pipeline performance.

Liu and Marrero [9] studied the coal-log pipeline transportation of Western Coal. The coal-log pipeline design and economics were studied and presented by Liu [10]. An extension of this work was made by Liu et al. [11] to study the economics

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of coal log pipeline for transporting coal. Liu and Marrero [12] presented an overview on the coal log pipeline, in which issues of hydrodynamics, effect of polymer additives, unsteady capsule flow, wear of coal logs, coal log manufacturing, economics and the state of development were highlighted.

Several researches were performed on modelling turbulent flow in annular space. Markatos et al. [13] provided mathematical analysis and numerical solutions for the flow in narrow skewed annuli to predict static pressure and velocity fields. Markatos et al. [14] studied and provided predictions of hydrodynamics and chemistry of confined turbulent methane–air flames in the annulus between two concentric tubes combustor to. Govier and Aziz [15] presented an overview for laminar and turbulent capsule flows in concentric and eccentric pipelines. They studied both cylindrical and spherical shapes of capsule. Garg [16] studied the effect of a non-uniform clearance over the capsule length taking into account the friction between the capsule and the pipe surfaces. Garner and Raithby [17] estimated the capsule velocity and velocity profiles for laminar eccentric capsule flow in the annulus.

Tomota and Fujiwara [18]analyzed the laminar flow capsule velocity and pressure drop across the capsule in hydraulic and pneumatic pipelines. Fujiwara et al. [19] and Tomita et al. 20] used the method of characteristics to study hydraulic capsule transport parameters such as the pressure drop, capsule velocity, capsule specific gravity, and the type of flow. Laminar–turbulent transition was numerically modeled by Ogawa et al. [21] to predict the velocity profile and pressure gradient in concentric annuli. The wake of capsule and the effect of interaction between two capsules on the drag were studied experimentally by Tsuji et al. [22].

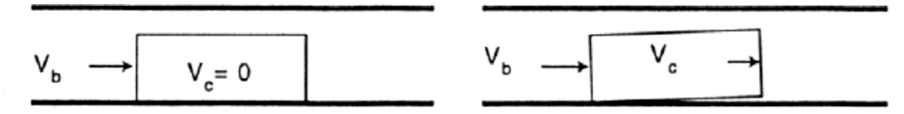
Sud and Chaddock [23] presented a numerical model for developing and fully developed flow in annulus. They reports drag calculations for vehicles in very long tubes. Polderman et al. [24] provided a numerical model for turbulent lubrication flow in an annular channel and measured experimentally the velocity profile and the pressure gradient in the annulus. Their numerical prediction was in good agreement with the experimental data.

Markatos [25], Ferziger and Peric [26] and Wilcox [27], considered comprehensively the turbulence modelling; its importance, its properties, the closure problem, the algebraic models, the one and two equation models, the effects of compressibility, beyond the Boussinesq approximation, numerical considerations, and the new research horizons such as large eddy simulation (LES), direct numerical simulation (DNS) models and the Reynolds stress (RSM) models.

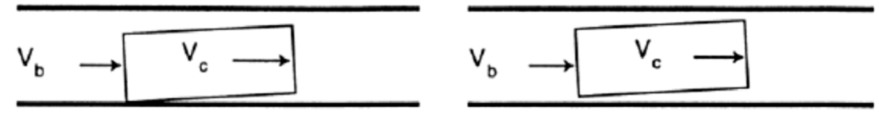
Swamee [28,29] extended the Kroonenberg's [30] mathematical model by including the expansion loss at capsule tail and the surface resistance loss in the inter-capsule distance. He derived an expression for the effective friction factor in which he integrated all types of losses. Also he performed a parametric study to obtain minimum power-loss configuration, cost considerations, optimization and design algorithm. Agarwal and Mishra [31] presented another design example and showed the variation of optimum diameter and length with cargo transport rate.

As described by Liu [3], the motion of capsules in a pipe can be classified into four regimes as shown in Fig. 1. In Regime 1, the bulk fluid velocity, $V_b$ is so low that insufficient drag is developed on the capsules to overcome the contact friction between the capsules and the pipe in order to move. Consequently, capsules of density higher that fluid rest on the pipe floor, whereas lighter density capsules rest against the pipe top. Regime 2 starts when the velocity of the fluid is high enough to cause the capsules slide along the pipe ($V_b > V_i$, where $V_i$ is the incipient velocity at which the capsule starts to slide). However, the fluid velocity in Regime 2 is still relatively low, the contact friction between the capsules and the pipe is high, and the capsule velocity, $V_c$ is less than the fluid velocity. By increasing the bulk velocity ($V_b$) beyond those in Regime 2, the pressure drop along the capsule (propelling force) is higher than the pressure drop of that of a flow free of capsules. Hence, the capsule velocity overtakes the fluid velocity. Also in Regime 3, the shear stress around the capsule (resisting force) is lower than the shear stress of the free of capsules. This is because the relative velocity between the flow in the annulus and the capsule wall is lower than the relative velocity between the bulk velocity and the pipe wall; the pipe wall is fixed and the capsule wall is moving in the same direction of the flow in the annulus. These two parameters cause the capsule to accelerate to reach a velocity higher than the bulk fluid velocity $V_b$ until the balance takes place between the pressure force and the shearing force on the capsule. This does not contradict with the continuity equation. The continuity equation will be

$$V_b \times \frac{\pi}{4} d_p^2 = V_c \times \frac{\pi}{4} d_c^2 + V_a \times \frac{\pi}{4}\left(d_p^2 - d_c^2\right),$$



(a) Regime 1 ( $V_b < V_i$ , $V_c = 0$ ) (b) Regime 2 ( $V_i < V_b < V_L$ , $V_c < V_b$ )



(c) Regime 3 ( $V_i < V_b < V_L$ , $V_c > V_b$ ) (d) Regime 4 ( $V_b > V_L$ , $V_c > V_b$ )

**Fig. 1.** Four regimes of hydraulic capsule pipeline flow [3].