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Using spectral element method for analyzing continuous beams and bridges subjected to a moving load

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ABSTRACT

Spectral element method in frequency domain is employed to analyze continuous beams and bridges subjected to a moving load. The formulation is developed for an Euler beam under a moving load with an arbitrary amplitude and velocity. It is shown that the procedure is simplified for a moving load with a constant amplitude and velocity. Static Green's function is used as a modifying function to improve the moment and shear force results. It is further shown that while modifying function is used in conjunction with spectral element method, fewer elements will be required to achieve proper results. The numerical examples show the accuracy of the method.

1. Introduction

Dynamic response of beams subjected to a moving load is an important issue in many engineering problems, especially in the design of bridges. In order to analyze vibration of structures under a moving load, Fryba [1] used integral transforms, particularly Laplace transformation. Olsson [2] used modal analysis with the exact mode shapes for analyzing a simply supported Euler beam under a moving load. Sun [3–5] proposed a closed form solution for infinite Euler beams and plates on different subgrades under a moving load. The load is characterized by Dirac-delta function and Fourier transform together with its inverse and convolution integral is used to find the structure's response. Kim [6] used fast Fourier transform (FFT) algorithm to investigate the response of an Euler beam under a moving load in the time domain. Kargarnovin and Younesian [7] used Fourier transformation to get the response of Timoshenko beams on the Pasternak subgrade. Garinei [8] used modal analysis and inspected the effects of velocity and some other parameters on the response of an Euler beam subjected to a harmonic moving load.

Although, integral transformations like Fourier or Laplace transforms are powerful tools for analyzing beams under moving loads, however, these methods are usually suitable for simple continuous beams. In reality, a bridge is made of different members and a more flexible method like the finite element method (FEM) is needed to better cope with the real problem. Wu et al. [9] used finite element technique to analyze a crane under a moving load. Anderson et al. [10], combined FEM and the boundary element method in order to analyze a 2D domain. Martinez-Castro et al. [11] proposed a new semi-analytic solution of non-uniform Bernoulli–Euler beams under a moving load. They used ordinary Hermitian shape functions for modeling the beam. Using the classical FEM for the spatial discretization imposes some errors in the calculation of the dynamic response [12–14]. Some of these errors cannot be eliminated by making the size of elements smaller [13]. On the other

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hand in order to evaluate the response of structures subjected to a load with high frequency contents, the size of the elements should be very small [12].

Recently, the FFT based spectral element method (SEM) [15–19] has been widely used in the dynamic analysis of structures. This method is based on solving the differential equations in the frequency domain for obtaining the shape functions of the element. These shape functions are utilized to evaluate the dynamic stiffness matrix which is used to calculate the responses of the systems subjected to arbitrary external loads in frequency domain. Subsequently, the results in the time domain are evaluated by using the inverse of FFT [18,19].

A thorough literature survey indicates that the SEM is not directly used to analyze frame structures like bridges which are subjected to a moving load with an arbitrary amplitude and speed. In the work of Henchi et al. [20] dynamic stiffness matrix is used to calculate the exact undamped mode shapes and the natural frequencies for continuous beams. They have used modal analysis and FFT method to solve modal equations and evaluate the response.

In the present paper, first the spectral element formulation is developed to analyze a bridge under a moving load, and then it is shown that through a modification, better results can be obtained. This modification greatly improves the accuracy of the shear force and the moment diagrams of the beams.

2. Theoretical formulation

The differential equation of motion of an Euler beam depicted in Fig. 1 is [18,19]

$$\rho A\frac{\partial^2 W}{\partial t^2}+\eta A\frac{\partial W}{\partial t}+EI\frac{\partial^4 W}{\partial \tilde{x}^4}=f(\tilde{x},t), \tag{1}$$

where W is the lateral displacement of the beam. ρ, A, E, I, η are the density, the area of the cross section of the beam, Young's modulus, the moment of inertia and viscous damping per unit volume, respectively. $f(\tilde{x},t)$ is an arbitrary external load. Using continuous Fourier transformation with zero initial conditions, Eq. (1) transforms to

$$(i\eta A\omega-\rho A\omega^2)\widehat{W}+EI\frac{\partial^4 \widehat{W}}{\partial \tilde{x}^4}=\hat{f}(\tilde{x},\omega), \tag{2}$$

where $i=\sqrt{-1}$, ω is frequency and $\widehat{W},\hat{f}$ are the lateral displacement and the loading function in the frequency domain. Using discrete Fourier transformation (Eq. (3)), Eq. (2) can be rewritten as Eq. (4) [18]

$$\widehat{W}(\tilde{x},\omega_n)=\Delta T\sum_{m=0}^{N-1}W_m(\tilde{x})e^{\frac{-2i\pi nm}{N}}, \tag{3}$$

$$(i\eta A\omega_n-\rho A\omega_n^2)\widehat{W}+EI\frac{\partial^4 \widehat{W}}{\partial \tilde{x}^4}=\hat{f}(\tilde{x},\omega_n), \tag{4}$$

where ΔT is the time interval for sampling, N is is the total number of points in the sampling and ω_n is frequency in discretized form. In the ordinary SEM, in order to evaluate shape functions, the homogeneous differential equation is solved. For a beam element with four degrees of freedom and length l, Fig. 1, the shape functions can be derived using Eq. (4) with the right side equal to zero [17–19].

In terms of nodal displacements, the lateral displacement is

$$\widehat{W}=\sum_{j=1}^{4}\widehat{N}_j\hat{w}_j, \tag{5}$$

where $\widehat{N}_j,\hat{w}_j$ are the shape functions and the nodal values in frequency domain.

The boundary conditions are

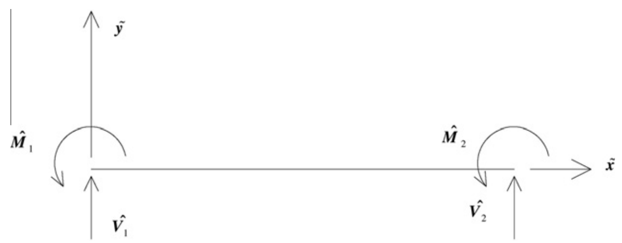

Fig. 1. Beam element and local degrees of freedom.