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The critical speed of a moving time-harmonic load acting on a system consisting a pre-stressed orthotropic covering layer and a pre-stressed half-plane

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ABSTRACT

The dynamic response of a system consisting of an initially stressed covering layer and an initially stressed half-plane to a moving time-harmonic load is investigated within the scope of the piecewise-homogeneous body model utilizing three-dimensional linearized wave propagation theory in the initially stressed body. It is assumed that the material of the layer and half-plane is orthotropic. It is also assumed that the velocity of the line-located time harmonic moving load which acts on the covering layer is constant. The investigations were carried out were for the plane-strain state under subsonic velocity of the moving load for two types of contact conditions, namely: complete and incomplete. An algorithm is developed for the determination of the values of the moving load's critical velocity. For various values of the problem parameters the numerical results were presented and discussed.

1. Introduction

Studies on dynamical responses of bodies to moving loads are great significance in both the theoretical and practical sense. The results of these investigations can be employed in many branches of modern transportation engineering applications, such as the design of track beds and bridges for high-speed trains, cars, trucks, etc., parking garages, ballistic systems such as track-based weapons, aircraft runways and so forth. A considerable part of these studies relates to the dynamical response of a layered half-plane to the moving load. In this case the layered half-plane is taken as the model to simulate track-based systems or various pavements subjected to moving vehicle loads. The relevant theoretical investigations in this field were made in a paper Achenbach et al. [1] and others listed therein. However, as time has elapsed, these studies were improved upon and developed continuously; the latest iterations are described in papers [2–5] and in many others.

In the aforementioned papers, the noted dynamical response was studied by employing the classical linear theory of elastodynamics and the equation of motion for the layer was described within the scope of Kirchhoff or Timoshenko beam-plate theories, but the equations of motion of the half-plane were described within the scope of the exact equations of the linear theory of elastic waves.

The present level of high speed transportation vehicle engineering requires investigations of problems of the foregoing type within the framework of more accurate theories that take nonlinear dynamical effects into account. An interesting and urgent problem which also applies to the nonlinear dynamical effects in the aforementioned elastic systems is the elastodynamics problem for initially stressed bodies. Initial stresses occur in the systems, covering layer and half-space after manufacturing and assembly and, in many cases, the magnitude of these stresses has a large effect.

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Under certain conditions, the type of problems noted above can be solved with the use of Three-dimensional Linearized Theory of Elastic Waves in the Initially Stressed Bodies (TLTEWISB). The constructions of the field equations of TLTEWISB and their applications to the wave propagation problem for initially stressed bodies are detailed in Guz [6] and elsewhere. A review of these investigations was presented in papers Guz [7] and Zhuk and Guz [8]. Furthermore, the time-harmonic stress field in layered pre-stretched bodies was studied in [9–13] and others by employing TLTEWISB.

Note that within the framework of TLTEWISB few studies have been done until now on the dynamical response of the pre-stressed layered half-space to the moving load; see for example [14,15]. In the paper Babich et al. [14], the dynamical response of the system consisting of the layer and pre-strained half-plane was considered. The equation of motion for covering layer was described by Timoshenko beam theory, but the equation of motion for the half-plane was described by TLTEWISB. The solution to the corresponding boundary value problem was determined by employing the exponential Fourier integral transformation. Concrete numerical investigations were made for the case where the constitutive relations for the half plane material were described using the harmonic type of potential. Moreover, it was assumed that the speed of the moving load is constant and the subsonic case has been taken into consideration. As a result of these numerical investigations, the influence of the problem parameters on the critical velocity was studied. In the paper Babich et al. [15], the foregoing problem was studied through the use of the complex potential of TLTEWISB.

In papers Akbarov et al. [16] and Akbarov and Ilhan [17] utilized the findings made in the papers [14,15] in developing the case where the covering layer also has the initial stress and the equation of motion for this layer has also been described by TLTEWISB. The influence of the problem parameters on the critical velocity was studied. However, in the paper Akbarov et al. [16] it was assumed that the materials of the covering layer and half-plane were isotropic. It is evident that this assumption significantly restricts the possibility of theoretical investigations regarding control of the values of the critical velocity of the moving load through the mechanical properties of the layer and half-plane materials. In the paper Akbarov and Ilhan [17] the investigation carried out, and in the paper Akbarov et al. [16] is further developed for the case where the materials of the covering layer and half-plane are anisotropic (orthotropic). Moreover, in the present paper it is assumed that the moving load is time-harmonic.

Throughout the investigations repeated indices indicated only in one side of the relations are summed over their ranges.

2. Formulation of the problem

Taking into consideration the initially stressed half-plane covered by the initially stressed layer, we determine the positions of the points of the layer and half-plane by the Lagrangian coordinates in the Cartesian system of coordinates $Ox_1x_2x_3$ (Fig. 1).

We assume that the layer and half-plane (before the contact) are stressed separately in the direction of the Ox_1 axis and in each of them the uni-axial homogeneous initial stress appears. Note that the covering layer and the half-plane occupy the regions $\{-\infty < x_1 < \infty, \quad -h \leqslant x_2 \leqslant 0, \quad -\infty < x_3 < +\infty\}$ and $\{-\infty < x_1 < +\infty, \quad -\infty \leqslant x_2 \leqslant -h, \quad -\infty < x_3 < \infty\}$, respectively (Fig. 1). The direction of the Ox_3 axis is perpendicular to the figure plan and therefore is not shown in Fig. 1.

The values related to the layer and half-plane are denoted by upper indices (1) and (2), respectively. Moreover, the values related to the initial stresses are denoted by the additional upper index 0.

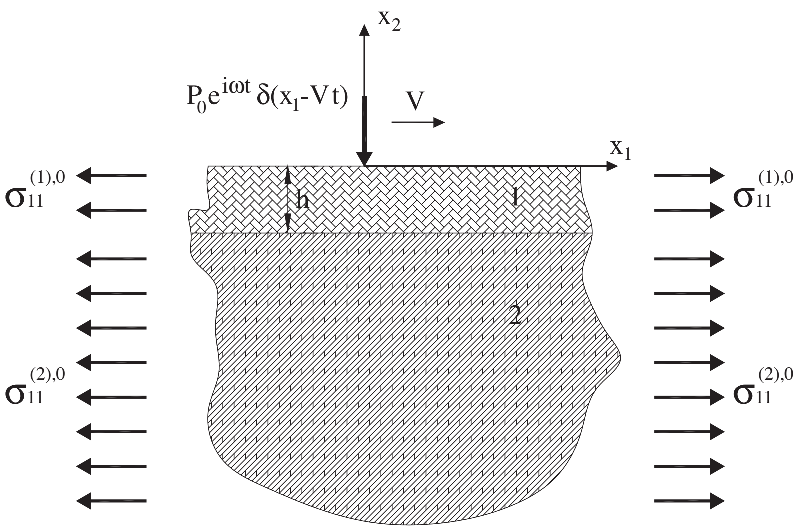

Fig. 1. The geometry of the structure of the half-plane covered by the layer.