Applied Mathematical Modelling 35 (2011) 1739–1750

Contents lists available at ScienceDirect

Applied Mathematical Modelling

journal homepage: www.elsevier.com/locate/apm

Static analysis of composite beams with weak shear connection

István Ecsedi, Attila Baksa *

Department of Mechanics, University of Miskolc, Miskolc-Egyetemváros H-3515, Hungary

ARTICLE INFO

Article history:
Received 11 July 2009
Received in revised form 21 September 2010
Accepted 4 October 2010
Available online 13 October 2010

Keywords:
Interlayer slip
Shear connection
Two-layered beam

ABSTRACT

The objective of the present paper is to analyse the static behaviour of elastic two-layer beams with interlayer slip. The Euler–Bernoulli hypothesis is assumed to hold for each layer separately, and a linear constitutive equation between the horizontal slip and the interlaminar shear force is considered. The applied loads act in the plane of symmetry of the composite beam, and the material and geometrical properties do not depend on the axial coordinate. Closed-form solutions for displacements and interlayer slips are developed. A second order differential equation is derived for the interlayer slip whose solution is used to determine the deflections and slopes. Examples illustrate the application of the method presented.

1. Introduction

Layered beams made of different linearly elastic materials are frequently used in the constructions and they create growing interest in different engineering sectors, where both high strength-to-weight and stiffness-to-weight ratio are required. There exist many ways to get connection between layers made by different materials. If the beam components are connected by means of strong bonds, the connection is considered as perfect in the sense that no displacement discontinuities occur at the interface between the components, so that this type of the layered beams can be modeled as a non-homogeneous beam [1,2]. In many other cases, the connection is weak in shear, permitting only the relative slip but preserving the contact in normal direction.

The pioneering work on laminated beams with weak shear connections was conducted by Newmark et al. [3] on a composite beam. The static analysis done by Newmark et al. [3] is based on the Euler–Bernoulli beam theory and become a basis of the subsequent investigations of layered beam systems with interlayer slip. A laminated beam theory with interlayer slip based on Timoshenko beam theory was developed by Murakami [4]. Exact first and second order static analyses for composite beam-columns with partial interaction subjected to transverse and axial loading was presented by Girhammar and Gupu [5]. Papers mentioned above give mainly closed form solutions for static bending problems of composite beam with weak shear connection. Thompson et al. [6] developed a finite element program to analyse the static behaviour of beam systems with weak shear connection. Two and three-field mixed finite element formulations for the linear and nonlinear analysis of partially connected composite beams are presented by Ayoub in [7] and Dall'Asta and Zona in [8].

Girhammar and Pan [9] derived, by using variational methods, the ordinary differential equations for the deflection and internal actions and all the pertaining admissible boundary conditions for partially composite Euler–Bernoulli beams and beam-columns. In paper by Girhammar and Pan [9] static loading conditions, including transverse and axial loading and first an second order analysis were considered. A simplified analysis and design method for composite members with partial interaction that predict the deflections and stresses has been proposed by Girhammar [10]. The presented approximate

* Corresponding author. Tel.: +36 46 565 111.
E-mail addresses: mechecs@uni-miskolc.hu (I. Ecsedi), mechab@uni-miskolc.hu (A. Baksa).

0307-904X/$ - see front matter
doi:10.1016/j.apm.2010.10.006

method in [10] is based on the concept of effective composite bending stiffness. The effective composite bending stiffness reflects the influence of the interlayer slip, and depends on the shear connector stiffness including slip modulus, cross-sectional material-geometrical properties and beam length [10].

The present paper deals with two layer beams with interlayer slip giving an analytical method. A slip-deflection formulation is developed which is based on a second order differential equation of the interlayer slip and the expression of bending moment in terms of slip and deflection. The problem is approached by assuming a kinematical model where the two beam components separately follow the Euler–Bernoulli hypothesis. A linear constitutive equation between the horizontal slip and interlaminar shear force is considered. The applied transverse loads act on the plane of symmetry of composite beam, and the geometrical and material properties are independent of the axial coordinate. This paper deals with only the plane problem of bending, where a symmetric two-layer beam undergoes vertical loads lying on its symmetry plane. Static equilibrium problems are considered. Solution is based on a system of differential equations whose equations can be solved step by step.

2. Governing equations

In the reference configuration the composite beam with two components occupies the cylindrical region $B = A \times (0,L)$ generated by translating its cross section A with a regular boundary ∂A along a rectilinear axis, normal to the cross section. The cross section A is divided into two parts A_1 and A_2 by a curve ∂A_{12} describing the position of continuous connection such that (Fig. 1)

$$B_i = A_i \times (0,L) \quad (i = 1,2), \tag{1}$$

$$A = A_1 \cup A_2, \quad B = B_1 \cup B_2, \tag{2}$$

$$\partial A_i = \partial A_{i0} \cup \partial A_{12} \quad (i = 1,2), \quad \partial A = \partial A_{10} \cup \partial A_{20}. \tag{3}$$

Here, L is the length of the beam. A point P in $\overline{B} = B \cup \partial B$ (∂B is the boundary surface of B) is indicated by the position vector $\boldsymbol{r} = x\boldsymbol{e}_x + y\boldsymbol{e}_y + z\boldsymbol{e}_z$, where x, y, z and $\boldsymbol{e}_x, \boldsymbol{e}_y, \boldsymbol{e}_z$ are referred to a rectangular coordinate system $Oxyz$. The z axis is located in the E-weighted center line of the whole fully composite beam, and the plane yz is the plane of symmetry for the geometrical and material properties and loading conditions. $\boldsymbol{e}_x, \boldsymbol{e}_y, \boldsymbol{e}_z$ are the unit vectors along the coordinate axis x, y and z, respectively. The center of A_i is denoted by C_i $(i = 1,2)$, and C is the E-weighted center of the whole cross section $A = A_1 \cup A_2$. The positions of C_1 and C_2 on an axis y with respect to $C \equiv O$ can be obtained as

$$c_1 = |\overrightarrow{CC_1}| = \frac{A_2E_2}{\langle AE \rangle}c, \quad c_2 = |\overrightarrow{CC_2}| = \frac{A_1E_1}{\langle AE \rangle}c, \tag{4}$$

$$c = c_1 + c_2 = |\overrightarrow{C_2C_1}|, \quad \langle AE \rangle = A_1E_1 + A_2E_2. \tag{5}$$

Here, E_i is the Young modulus of beam component B_i with cross section A_i $(i = 1,2)$, and we remark that, the y coordinate of C_1 is positive $(y_1 = c_1)$ and the y coordinate of C_2 is negative $(y_2 = -c_2)$ (Fig. 1).

According to the Euler–Bernoulli hypothesis (kinematical assumption) which is valid for each homogeneous individual beam components the deformed configuration is described by the displacement field $\boldsymbol{u} = u(x,y,z)\boldsymbol{e}_x + v(x,y,z)\boldsymbol{e}_y + w(x,y,z)\boldsymbol{e}_z$ which has the form

$$u = 0 \quad \text{in} \quad \overline{B}, \tag{6}$$

$$v = v(z) \quad \text{in} \quad \overline{B}, \tag{7}$$

$$w = w(y,z) = w_i(z) - y\frac{\mathrm{d}v}{\mathrm{d}z} \quad (x,y,z) \in B_i \quad (i = 1,2). \tag{8}$$

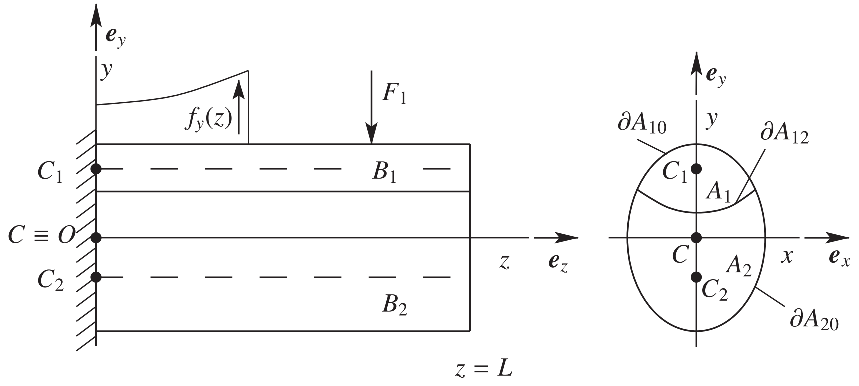

Fig. 1. Two-layered beam with weak shear connection.