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A method for group decision-making based on determining weights of decision makers using TOPSIS

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ABSTRACT

In general, weights of decision makers (DMs) play a very important role in multiple attribute group decision-making (MAGDM), how to measure the weights of DMs is an interesting research topic. This paper presents a new approach for determining weights of DMs in group decision environment based on an extended TOPSIS (Technique for Order Preference by Similarity to an Ideal Solution) method. We define the positive ideal solution as the average of group decision. The negative ideal solution includes two parts: left and right negative ideal solution, which are the minimum and maximum matrixes of group decision, respectively. We give an example to illustrate the developed approach. Finally, the advantages and disadvantages of this study are also compared.

1. Introduction

Multiple attribute decision-making (MADM) is an important part of modern decision science. It always involves multiple decision attributes and multiple decision alternatives. The purpose of the decision-making is finding the most desirable alternative(s) from a discrete set of feasible alternatives with respect to a finite set of attributes. It has been extensively applied to various areas such as society, economics, military, management, etc. [1–6], and has been receiving more and more attention over the last decades [7,8].

The increasing complexity of the socio-economic environment makes it less and less possible for single decision maker (DM) to consider all relevant aspects of a problem. As a result, many decision-making processes, in the real world, take place in group settings [9]. Moving from single DM's setting to group members' setting introduces a great deal of complexity into the analysis. For example, consider that these DMs usually come from different specialty fields, and thus each DM has his/her unique characteristics with regard to knowledge, skills, experience and personality, which implies that the DM usually has different influence in overall decision result. That is, the weights of DMs are different. Therefore, how to determine the weights of DMs will be an interesting and important research topic. At present, many methods have been proposed to determine the weights of DMs, for example, French [10] proposed a method to determine the relative importance of the groups members by using the influence relations, which may exist between the members. Theil [11] proposed a method based on the correlation concepts when the member's inefficacy is measurable. Keeney and Kirkwood [12], and Keeney [13] suggested the use of interpersonal comparisons to obtain the values of scaling constants in the weighted additive social choice function. Bodily [14] derived the member weight as a result of designation of voting weights from a member to a delegation subcommittee made up of other members of the group. By using the deviation measures between additive linguistic preference

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relations, Xu [15] gave a straightforward method to determine the weights of DMs by Bodily' method [14]. Mirkin and Fishburn [16] proposed two approaches which use the eigenvectors method to determine the relative importance of the group's members. Martel and Ben Khélifa [17] proposed a method to determine the relative importance of groups members by using individual outranking indexes. Van den Honert [18] used the REMBRANDT system (multiplicative AHP and associated SMART model) to quantify the decisional power vested in each member of a group, based on subjective assessments by the other group members. Jabeur and Martel [19] proposed a procedure which exploits the idea of Zeleny [3] to determine the relative importance coefficient of each member. Brock [20] used a Nash bargaining based approach to estimating the weights of group members intrinsically. Ramanathan and Ganesh [21] proposed a simple and intuitively appealing eigenvector based method to intrinsically determine the weights of group members using their own subjective opinions. Chen and Fan [22] proposed a factor score method for obtaining a ranking of the assessment levels of experts in group-decision analysis. Yue [23] developed a method for determining weights of DMs with interval numbers. In this article, we shall discuss the weights of DMs based on the Technique for Order Preference by Similarity to an Ideal Solution (TOPSIS).

TOPSIS, the well-known classical MADM methods, was first developed by Hwang and Yoon [24]. It helps DMs organizing the problems to be solved, and carry out analysis, comparing and rankings of the alternatives. Accordingly, the selection of a suitable alternative(s) will be made.

The basic idea of TOPSIS is rather straightforward. It simultaneously considers the distances to both positive ideal solution (PIS) and negative ideal solution (NIS), and a preference order is ranked according to their relative closeness, and a combination of these two distance measures [24–32]. That is, the best alternative has simultaneously the shortest distance from the PIS and the farthest distance from the NIS. The PIS is identified with a "hypothetical alternative" that has the best values for all considered attributes whereas the NIS is identified with a "hypothetical alternative" that has the worst attribute values.

The existing approaches have significant contributions to solving the weights of DMs problems. Most of the literature mentioned above described the individual decision information by a multiplicative preference matrix. Until now there has been little investigation of the weights of DMs based on individual decision information, in which the attribute values are given as observations in nonnegative real numbers, and the DMs have their subjective preferences on alternatives. The aim of this paper is to propose a novel approach to determining the weights of DMs. The extended TOPSIS technique is also called group TOPSIS in this article. For the given individual decision matrixes, the PIS of group opinion is depicted by a matrix, in which elements are expressed in average of all individual decisions. The NIS includes two parts: left and right negative ideal solutions, which are also depicted by a matrix, respectively. Specifically, for the normalized group decision matrixes, the left negative ideal solution (L-NIS) is the minimum matrix of group decision matrixes, the right negative ideal solution (R-NIS) is the maximum matrix of group decision matrixes, and both are expressed in maximum separation from the PIS.

The paper is organized as follows. In the next section, briefly introduces the traditional TOPSIS and multiple attribute group decision-making (MAGDM) method. In Section 3, we present an algorithm to determine weights of DMs based on an extended TOPSIS method. In Section 4, we illustrate our proposed algorithmic method with an example. In Section 5, we compare the proposed method with other methods. The final section concludes.

2. TOPSIS method and MAGDM problems

In this section, we review the TOPSIS method and MAGDM problems.

For convenience, we first let $M = \{1,2,\ldots,m\}$, $N = \{1,2,\ldots,n\}$ and $T = \{1,2,\ldots,t\}$; $i \in M$, $j \in N$, $k \in T$. Let $A = \{A_1,A_2,\ldots,A_m\}$ $(m \geqslant 2)$ be a discrete set of m feasible alternatives, $U = \{u_1,u_2,\ldots,u_n\}$ be a finite set of attributes, and $D = \{d_1,d_2,\ldots,d_t\}$ be a group of DMs, and $\lambda = (\lambda_1,\lambda_2,\ldots,\lambda_t)^T$ is the weight vector of DMs, where $\lambda_k \geqslant 0,\ \sum_{k=1}^{n}\lambda_k = 1$.

2.1. Representation of TOPSIS method

For a MADM problem, suppose each alternative is evaluated with respect to the n attributes, whose values constitute a decision matrix denoted by

$$X = (x_{ij})_{m\times n} = \begin{array}{c} \\ A_1 \\ A_2 \\ \vdots \\ A_m \end{array} \begin{array}{c} \begin{array}{cccc} u_1 & u_2 & \cdots & u_n \end{array} \\ \begin{pmatrix} x_{11} & x_{12} & \cdots & x_{1n} \\ x_{21} & x_{22} & \cdots & x_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ x_{m1} & x_{m2} & \cdots & x_{mn} \end{pmatrix} \end{array}. \tag{1}$$

The TOPSIS method (see Fig. 1) consists of the following steps [32,33]:

1. Normalize the decision matrix.
   In general, there are benefit attributes and cost attributes in the MADM problems. In order to measure all attributes in dimensionless units and facilitate inter-attribute comparisons, we introduce the following formulas to normalize each attribute value x_{ij} in decision matrix $X = (x_{ij})_{m\times n}$ into a corresponding element r_{ij} in normalized decision matrix given by Eq. (2).